education

Department:
Education
PROVINCE OF KWAZULU-NATAL

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS

COMMON TEST

MARCH 2020

MARKS: 100

TIME: 2 hours

N.B. This question paper consists of 8 pages and an information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 8 questions.

2. Answer **ALL** questions.

3. Clearly show **ALL** calculations, diagrams, graphs, et cetera that you have used in determining your answers.

4. Answers only will not necessarily be awarded full marks.

5. An approved scientific calculator (non-programmable and non-graphical) may be used, unless stated otherwise.

6. If necessary, answers should be rounded off to TWO decimal places, unless stated otherwise.

7. Diagrams are NOT necessarily drawn to scale.

8. Number the answers correctly according to the numbering system used in this question paper. Write neatly and legibly.

QUESTION 1

Given the quadratic sequence: $p;\ 5;\ q;\ 19;\ \ldots$

1.1 Calculate the value(s) p and q if the second constant difference is 2. (4)

1.2 Determine the n^{th} term of the quadratic sequence. (4)

1.3 Determine the first term of the sequence that will have a value greater than 10301. (4)

[12]

QUESTION 2

The sum to n terms of an arithmetic sequence is 36. The first and last terms are 1 and 11 respectively. Determine the number of terms n and its common difference d.

[5]

QUESTION 3

3.1 In a geometric sequence the first term is "a" and its common ratio is "r". Prove that the sum to n terms of the sequence is

$$S_n = \frac{a(1-r^n)}{1-r}$$

(4)

3.2 Given:

$$\sum_{k=1}^{m} 3.2^{1-k}$$

3.2.1 Write down the first TWO terms of the geometric sequence. (2)

3.2.2 Calculate the value of m if

$$\sum_{k=1}^{m} 3.2^{1-k} = \frac{3069}{512}$$

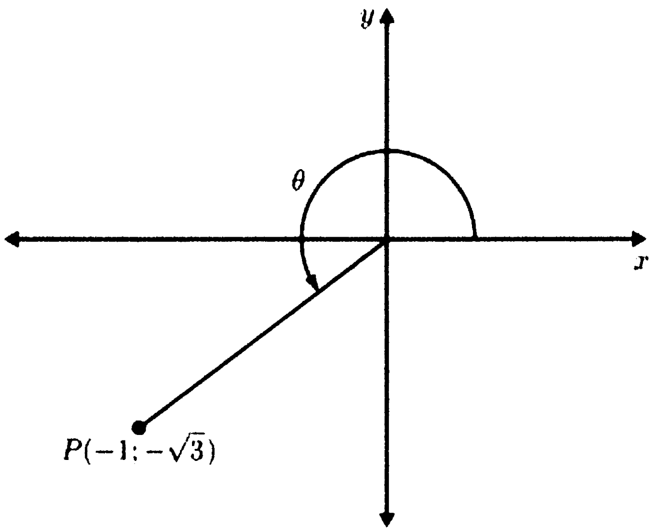
(5)

3.3 A ball is dropped from a height of 24 metres. Every time the ball hits the ground, it rises $\frac{3}{4}$ of its original height. Calculate the sum of the total vertical heights reached by the ball after the first bounce until it comes to rest. (3)

[14]

QUESTION 4

4.1 Use the diagram below to calculate, **without the use of a calculator**, the value of each of the following:



4.1.1 $\sin(-\theta)$ (3)

4.1.2 $\cos 2\theta$ (4)

4.2 **Without using a calculator**, determine the value of the following expressions.

4.2.1 $\sin 75° \cos 45° - \sin 15° \cos 45°$ (5)

4.2.2 $\dfrac{\cos 100°}{\sin(-10°)} \times \tan^2 120°$ (6)

4.3

4.3.1 Prove that, for angles α and β.

$$\frac{\sin \alpha}{\sin \beta} - \frac{\cos \alpha}{\cos \beta} = \frac{2\sin(\alpha - \beta)}{\sin 2\beta}$$ (4)

4.3.2 Hence, or otherwise, show that:

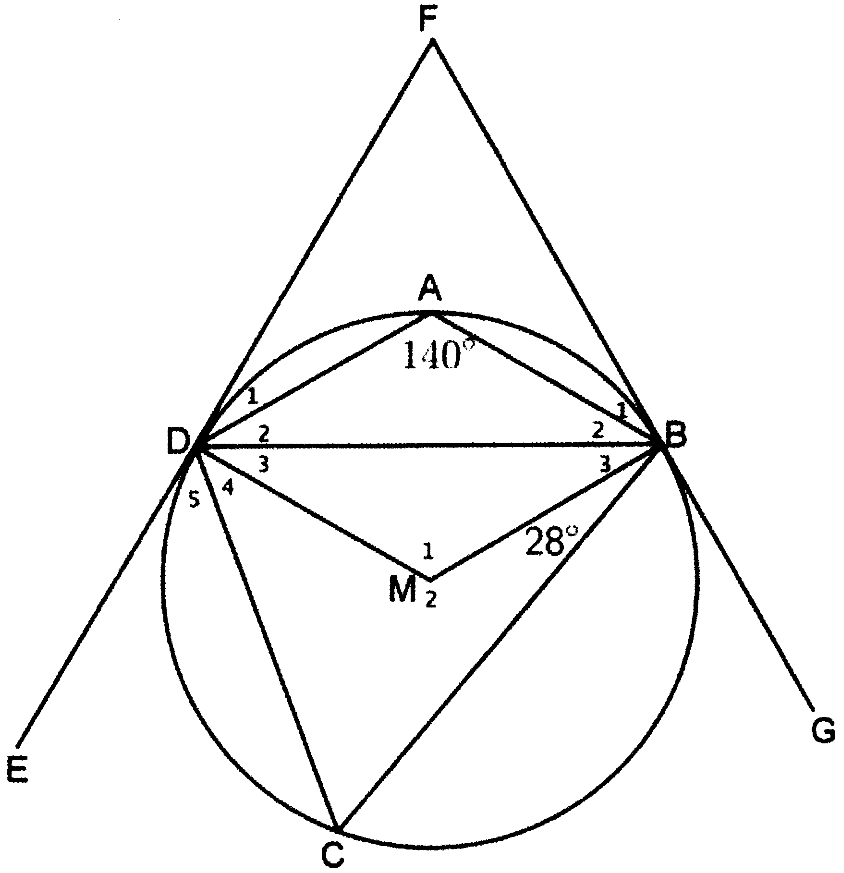
$$\frac{\sin 5\beta}{\sin \beta} - \frac{\cos 5\beta}{\cos \beta} = 4\cos 2\beta$$ (3)

4.4 Determine the general solution of:
$2\sin^2 x - \sin x - 1 = 0$ (5)

[30]

QUESTION 5

In the diagram below, M is the centre of the circle DABC. EDF is a tangent to the circle at D and FBG is another tangent to the circle at B.



Calculate the following angles, with reasons:

5.1 $\hat{C}$ (2)

5.2 $\hat{M}_1$ (3)

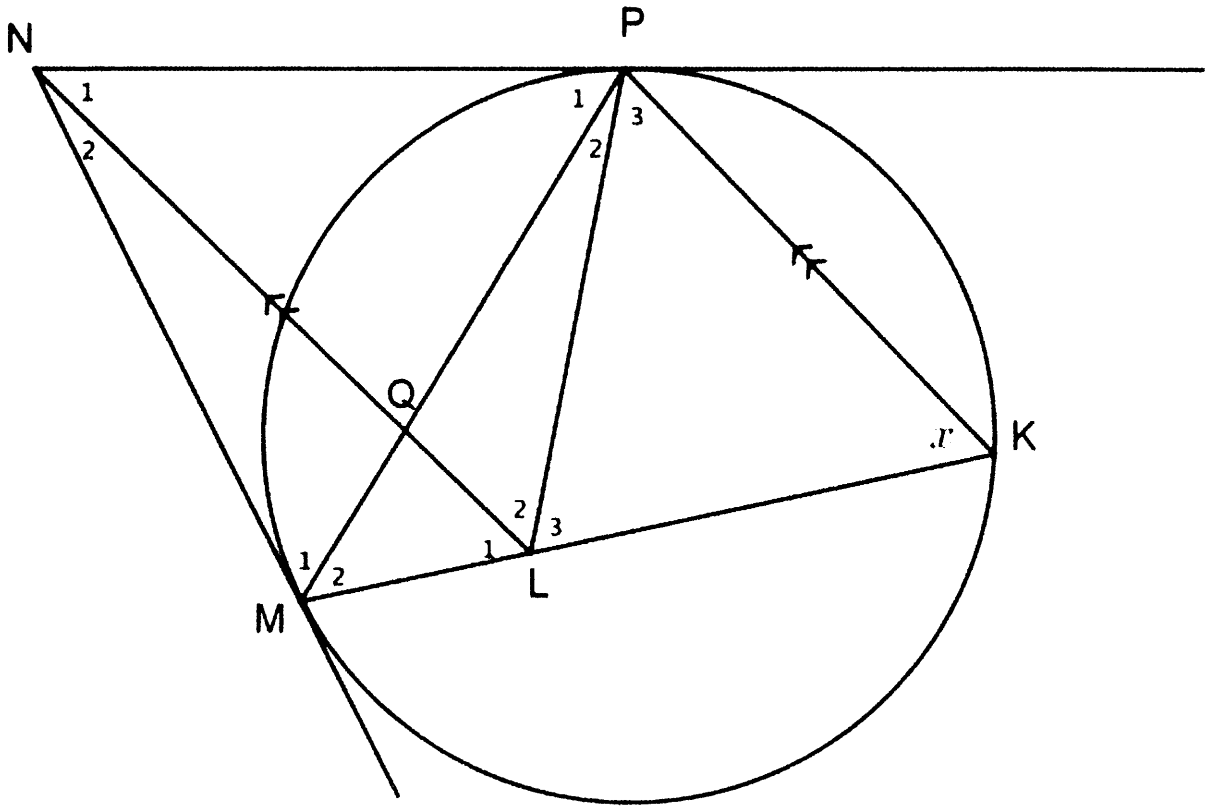
5.3 $\hat{B}_3$ (3)

5.4 $\hat{D}_5$ (2)

[10]

QUESTION 6

In the diagram below, NP and NM are tangents to the circle at P and M. K is a point on the circle KP, KM and PM are chords so that $\widehat{K} = x$. L is a point on KM so that KP//LN, L and P are joined.



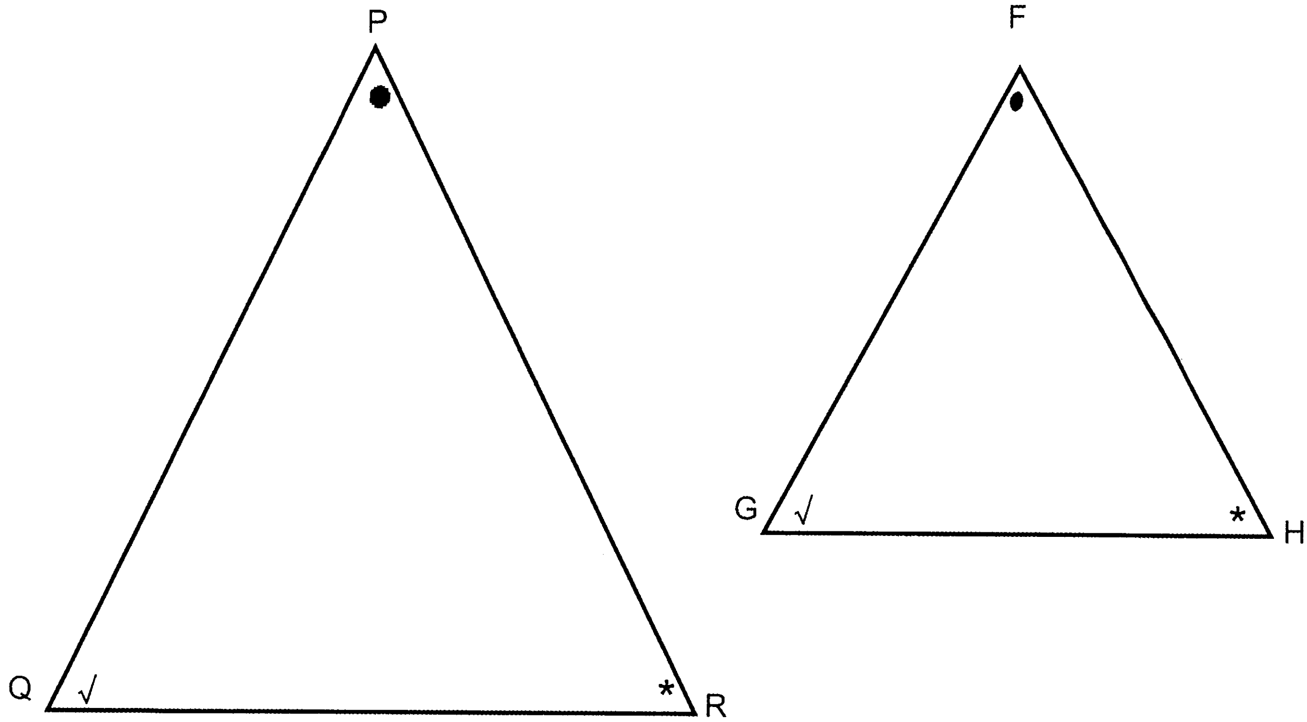
6.1 Prove that NMLP is a cyclic quadrilateral. (4)

6.2 Prove that ΔKLP is isosceles. (6)

[10]

QUESTION 7

ΔPQR and ΔFGH are given, $\hat{P}=\hat{F};\hat{Q}=\hat{G}$ and $\hat{R}=\hat{H}$



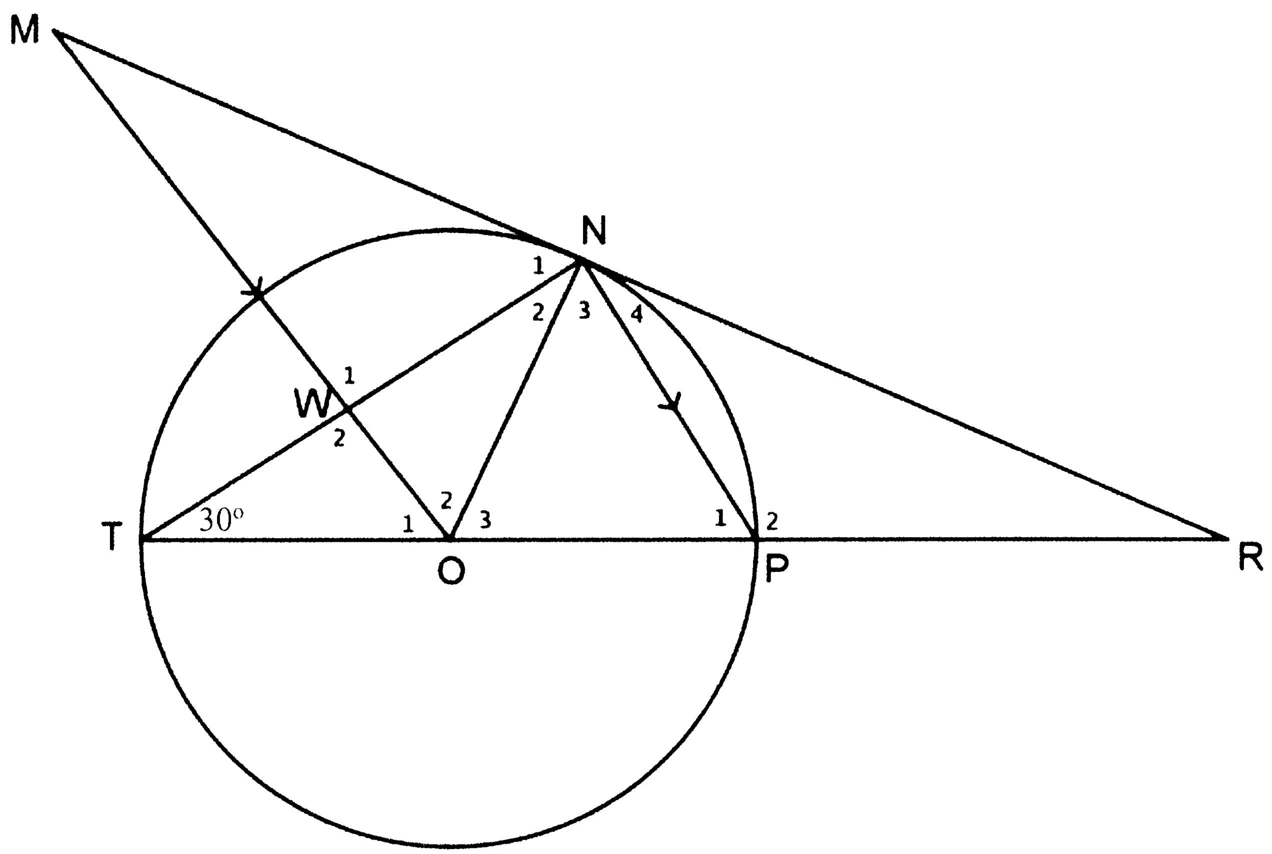
Prove theorem which states that if ΔPQR and ΔFGH are equiangular, then

$$\frac{PQ}{FG}=\frac{PR}{FH}$$

[6]

QUESTION 8

In the diagram below, TP is a diameter in the circle with centre O. TP is extended to R. RM is a tangent to the circle at N. MO intersects chord NT at W. NP//MO. $W\hat{T}O = 30^\circ$



8.1 Give, with reasons, THREE other angles each equal to 30°. (3)

8.2 Determine $R\hat{N}T$ (2)

8.3 Prove that:

8.3.1 ΔRNP /// ΔRTN (3)

8.3.2 $TW.\ RN = \frac{1}{2}RT.NP$ (5)

[13]

TOTAL MARKS: 100

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $A = P(1-ni)$ $A = P(1-i)^n$ $A = P(1+i)^n$

$T_n = a + (n-1)d$ $S_n = \frac{n}{2}[2a + (n-1)d]$

$T_n = ar^{n-1}$ $S_n = \frac{a(r^n - 1)}{r - 1}$; $r \neq 1$ $S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x[(1+i)^n - 1]}{i}$ $P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $y - y_1 = m(x - x_1)$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $m = \tan\theta$

$(x-a)^2 + (y-b)^2 = r^2$

InΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$$a^2 = b^2 + c^2 - 2bc.\cos A$$

$$area\ \Delta ABC = \frac{1}{2}ab.\sin C$$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$ $\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum x}{n}$ $\sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$ $P(A) = \frac{n(A)}{n(S)}$

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ $\hat{y} = a + bx$

$$b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$$

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MARKING GUIDELINES

MARKS: 100

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This memorandum consists of 10 pages

QUESTION 1

1.1	p 5 q 19 1D $5-p$ $q-5$ $19-q$ $q+p-10$ $24-2q$ 2D 2 2 $\boldsymbol{24-2q=2}$ $\boldsymbol{q=11}$ $\boldsymbol{q+p-10=2}$ $\boldsymbol{p=1}$	A✓ $24-2q=2$ CA✓q value A✓$q+p-10=2$ CA✓p value	(4)
1.2	$\boldsymbol{2a=2 \quad \therefore a=1}$ $\boldsymbol{3a+b=4 \quad \therefore b=1}$ $\boldsymbol{a+b+c=1 \quad \therefore c=-1}$ $\boldsymbol{T_n=n^2+n-1}$ **OR** $\boldsymbol{2a=2 \quad \therefore a=1}$ $\boldsymbol{3a+b=4 \quad \therefore b=1}$ $\boldsymbol{\therefore c=T_0=-1}$ $\boldsymbol{T_n=n^2+n-1}$	A✓a – value CA✓ b – value CA✓ c – value CA✓n^{th} term **OR** A✓a – value CA✓ b – value CA✓ c – value CA✓n^{th} term	(4) (4)
1.3	$\boldsymbol{T_n=n^2+n-1=10301}$ $\boldsymbol{n^2+n-10302=0}$ $\boldsymbol{(n+102)(n-101)=0}$ $\boldsymbol{n=-102 \quad or \quad n=101}$ $\boldsymbol{102^{2nd}}$ term **OR** $T_n=n^2+n-1>10301$ $n^2+n-10302>0$ $(n+102)(n-101)>0$ $n<-102 \quad or \quad n>101$ 102^{2nd} term	CA✓equating n^{th} term to 10301 CA✓ factors/quadratic formula CA✓ values of n CA✓102 **OR** CA✓setting up inequality CA✓ factors/quad. formula CA✓ interval and notation CA✓102	(4) (4)
			[12]

QUESTION 2

	$S_n = \frac{n}{2}[a + T_n]$ $36 = \frac{n}{2}[1 + 11]$ $36 = 6n$ $6 = n$ $11 = a + (n-1)d$ $11 = 1 + (6-1)d$ $2 = d$ **OR** $S_n = \frac{n}{2}[2a + (n-1)d]$ $36 = \frac{n}{2}[2(1) + (n-1)d] \quad \rightarrow \quad (1)$ $11 = a + (n-1)d$ $11 = 1 + (n-1)d$ $10 = (n-1)d \quad \rightarrow \quad (2)$ $36 = \frac{n}{2}[2(1) + 10]$ $72 = 12n$ $n = 6$ $10 = (6-1)d$ $2 = d$	A✓substituting into sum formula CA✓$36 = 6n$ CA✓ n – value CA✓substituting into general term formula CA✓ d – value **OR** A✓substituting into sum formula and forming equation (1) A✓substituting into general term formula and forming equation (2) CA✓$72 = 12n$ CA✓ n – value CA✓ d – value	(5) (5)

QUESTION 3

3.1	$S_n = a + ar + \cdots + ar^{n-1} \quad \to (1)$ $rS_n = ar + ar^2 + \cdots + ar^{n-1} + ar^n \quad \to (2)$ (2) – (1): $rS_n - S_n = ar^n - a$ $S_n(r^n - 1) = a(r^n - 1)$ $S_n = \frac{a(r^n-1)}{r-1}$	A✓writing equation A✓multiplying all terms by r A✓subtracting: LHS and RHS A✓factorizing	(4)
3.2.1	$3\ ;\ \frac{3}{2}$	AA✓✓each value	(2)
3.2.2	$S_m = \frac{3\left[(1-\left(\frac{1}{2}\right)^m\right]}{1-\frac{1}{2}} = \frac{3069}{512}$ $6\left[1-\left(\frac{1}{2}\right)^m\right] = \frac{3069}{512}$ $\left[1-\left(\frac{1}{2}\right)^m\right] = \frac{1023}{1024}$ $\left(\frac{1}{2}\right)^m = \frac{1}{1024}$ $\left(\frac{1}{2}\right)^m = \left(\frac{1}{2}\right)^{10}$ $m = 10$	CA✓substituting into sum formula CA✓ $\left[1-\left(\frac{1}{2}\right)^m\right] = \frac{1023}{1024}$ CA✓$\left(\frac{1}{2}\right)^m = \frac{1}{1024}$ CA✓expressing 1024 as a power of 2 CA✓ m – value N.B. Can be solved using logs.	(5)
3.3	$a = 24\left(\frac{3}{4}\right) = 18 \quad and \quad r = \frac{3}{4}$ $S_\infty = \frac{a}{1-r}$ $= \frac{24\left(\frac{3}{4}\right)}{1-\frac{3}{4}}$ $= 72\ m$	A✓ r – value CA✓ substituting into sum to infinity formula CA✓ answer	(3)
			[14]

QUESTION 4

4.1

4.1.1	$r^2 = x^2 + y^2$ $= (-1)^2 + (-\sqrt{3})^2$ $= 1 + 3$ $= 4$ $\therefore r = 2$ $sin(-\theta) = -sin\,\theta$ $= -\left(\frac{-\sqrt{3}}{2}\right)$ $= \frac{\sqrt{3}}{2}$ [Box: If $sin(-\theta) = sin\,\theta$ $= -\frac{\sqrt{3}}{2}$ Max. 2/3]	A✓ $r = 2$ CA✓ substitution CA✓ answer	(3)
4.1.2	$\cos 2\theta = 1 - 2\,sin^2\,\theta$ $= 1 - 2\left(\frac{-\sqrt{3}}{2}\right)^2$ $= 1 - 2\left(\frac{3}{4}\right)$ $= \left(1 - \frac{3}{2}\right) = -\frac{1}{2}$ **OR** $\cos 2\theta = 2\,cos^2\,\theta - 1$ $= 2\left(\frac{-1}{2}\right)^2 - 1$ $= 2\left(\frac{1}{4}\right) - 1$ $= \frac{1}{2} - 1 = -\frac{1}{2}$ **OR** $cos2\theta = cos^2\theta - sin^2\theta$ $= \left(\frac{-1}{2}\right)^2 - \left(\frac{-\sqrt{3}}{2}\right)^2$ $= \frac{1}{4} - \frac{3}{4}$ $= -\frac{1}{2}$	A✓ $1 - 2\sin^2\theta$ CA✓ substitution into the correct expression CA✓ simplification CA✓ answer **OR** A✓ $\cos 2\theta = 2\cos^2\theta - 1$ CA✓ substitution into the correct expression CA✓simplification CA✓answer **OR** A✓ $\cos 2\theta = \cos^2\theta - \sin^2\theta$ CA✓ substitution into the correct expression CA✓ simplification CA✓ answer	(4) (4) (4)

4.2			
4.2.1	$\sin 75^o \cos 45^o - \sin 15^o \cos 45^o$ $= \sin(90^o - 15^o).\cos 45^o - \sin 15^o \sin 45^o$ $= \cos 15^o \cos 45^o - \sin 15^o \sin 45^o$ $= \cos(45^o + 15^o)$ $= \cos 60^o$ $= \frac{1}{2}$ **OR** $\sin 75^o \cos 45^o - \sin 15^o \cos 45^o$ $\cos 45^\circ(\sin 75^\circ - \sin 15^\circ)$ Now $\sin 75^\circ - \sin 15^\circ$ $= \sin(45^\circ + 30^\circ) - \sin(45^\circ - 30^\circ)$ $= 2\cos 45^\circ \sin 30^\circ$ $\cos 45^\circ(\sin 75^\circ - \sin 15^\circ)$ $= \cos 45^\circ . 2\cos 45^\circ \sin 30^\circ$ $= \frac{\sqrt{2}}{2}.2.\frac{\sqrt{2}}{2}.\frac{1}{2}$ $= \frac{1}{2}$	A✓ $\sin 75^o = \cos 15^o$ A✓ $\cos 45^o = \sin 45^o$ CA✓$\cos(45^o + 15^o)$ CA✓ $\cos 60^o$ CA✓ $\frac{1}{2}$ (ACCEPT 0,5) **OR** ✓removing common factor ✓$\sin 75^\circ - \sin 15^\circ = 2\cos 45^\circ \sin 30^\circ$ ✓ $\cos 45^\circ = \frac{\sqrt{2}}{2}$ ✓$\sin 30^\circ = \frac{1}{2}$ ✓answer	(5) (5)
4.2.2	$\frac{\cos 100^o}{\sin(-10^o)} \times \tan^2 120^o$ $= \frac{\cos(90^o + 10^o)}{-\sin 10^o} \times \tan^2 60^o$ $= \frac{-\sin 10^o}{-\sin 10^o} \times (\sqrt{3})^2$ $= 3$ **OR** $\frac{\cos 100^o \times \tan^2 120^o}{\sin(-10^o)}$ $= \frac{(-\cos 80^o) \times (-\tan 60^o)^2}{(-\sin 10^o)}$ $= \frac{(-\sin 10^\circ) \times (-\sqrt{3})^2}{-\sin 10^\circ}$ $= 3$	✓$\cos(90^o + 10^o)$ ✓ $-\sin 10^\circ$ ✓ $\tan^2 60^\circ$ ✓ $-\sin 10^\circ$ ✓ $\sqrt{3}$ ✓3 **OR** ✓$-\cos 80^o$;✓$-\sin 10^\circ$; ✓$\tan^2 60^\circ$ ✓$-\cos 80^o = -\sin 10^o$; ✓$-\sqrt{3}$ ✓3	(6) (6)

4.3.1	LHS $= \frac{\sin\alpha}{\sin\beta} - \frac{\cos\alpha}{\cos\beta}$ $\frac{\sin\alpha\cos\beta - \cos\alpha\sin\beta}{\sin\beta\cos\beta}$ $= \frac{\sin(\alpha-\beta)}{\sin\beta\cos\beta}$ $= \frac{2}{2} \times \frac{\sin(\alpha-\beta)}{\sin\beta\cos\beta}$ $= \frac{2\sin(\alpha-\beta)}{2\sin\beta\cos\beta}$ $= \frac{2\sin(\alpha-\beta)}{\sin 2\beta}$ = RHS	A✓writing as a single fraction A✓$\sin(\alpha-\beta)$ A✓$\frac{2}{2}$ A✓simplification	(4)
4.3.2	$\frac{\sin 5\beta}{\sin\beta} - \frac{\cos 5\beta}{\cos\beta} = 4\cos 2\beta$ $LHS = \frac{\sin 5\beta}{\sin\beta} - \frac{\cos 5\beta}{\cos\beta}$ $= \frac{\sin 5\beta\cos\beta - \cos 5\beta\sin\beta}{\sin\beta\cos\beta}$ $= \frac{\sin(5\beta-\beta)}{\sin\beta\cos\beta}$ $= \frac{\sin 4\beta}{\frac{1}{2}.2\sin\beta\cos\beta}$ $= \frac{2\sin 2\beta\cos 2\beta}{\frac{1}{2}\sin 2\beta}$ $= 4\cos 2\beta$ = RHS **OR** Replace $\alpha = 5\beta$ in 4.3.1 $RHS = \frac{2\sin(5\beta-\beta)}{\sin 2\beta}$ $= \frac{2\sin 4\beta}{\sin 2\beta}$ $= \frac{2.2\sin 2\beta\cos 2\beta}{\sin 2\beta}$ $= 4\cos 2\beta$	A✓writing as a single fraction A✓$\sin 4\beta = 2\sin 2\beta\cos 2\beta$ A✓$\frac{1}{2}\sin 2\beta$ **OR** ✓ $\alpha = 5\beta$ ✓ $\sin 4\beta$ ✓expansion $\sin 4\beta$	(3) (3)

4.4	$2\sin^2 x - \sin x - 1 = 0$ $(2\sin x + 1)(\sin x - 1) = 0$ $\sin x = -\frac{1}{2}$ *or* $\sin x = 1$ $\sin x = -\frac{1}{2}$ $x = 210^\circ + k.360^\circ$ *or* $x = 330 + k.360; k \in Z.$ **OR** $\sin x = 1$ $x = 90^\circ + k.360^\circ; k \in Z$	A✓ factorization CA✓ $\sin x = -\frac{1}{2}$; $\sin x = 1$ CA✓ $x = 210^\circ + k.360^\circ; k \in Z$ CA✓ $x = 330^\circ + k.360^\circ; k \in Z$ CA✓ $x = 90^\circ + k.360^\circ; k \in Z$	**[5]**

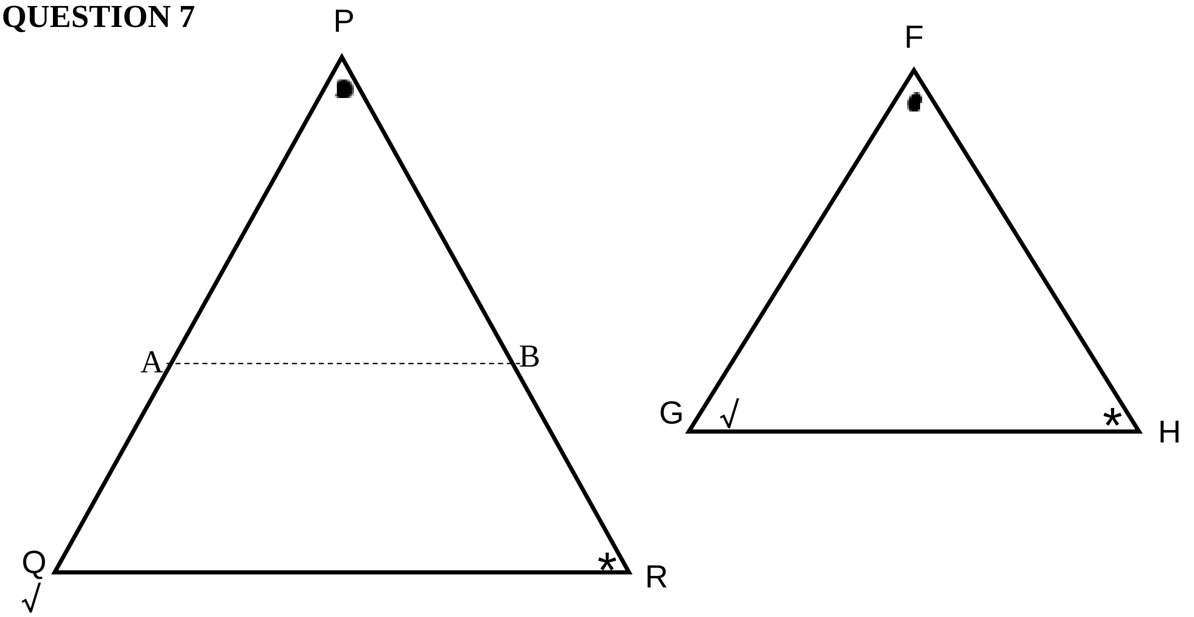
QUESTION 5

5.1	$\hat{C} + 140^\circ = 180^\circ$ opp ∠s of cyclic quad $\therefore \hat{C} = 40^\circ$	✓ S/R ✓ A Answer only with reason 2/2	(2)
5.2	$\hat{M}_1 = 2\hat{C}$ ∠ at centre is twice ∠ at circum. $= 2(40^\circ)$ $= 80^\circ$	✓ S✓R 80° ✓ CA Answer only with reason 3/3	(3)
5.3	$\hat{B}_3 = \frac{1}{2}(180^\circ - 80^\circ)$ ∠s opp = sides $= 50^\circ$	✓ S✓R ✓ A Answer only with reason 3/3	(3)
5.4	$\hat{D}_5 = \hat{B}_3 + 28^\circ$ tan − chord theorem $= 50^\circ + 28^\circ$ $= 78^\circ$	✓ S/R ✓ CA Answer Answer only with reason 2/2	(2)
			[10]

QUESTION 6

6.1	$\hat{L}_1 = x$ corr s ; KP// LN $\hat{P}_1 = x$ Tan-chord theorem $\hat{L}_1 = \hat{P}_1$ NMLP is a cyclic quad (Converse s in the same segment)	S✓R ✓S/R ✓R	(4)
6.2	$\hat{M}_1 = x$ Tan-chord theorem $\hat{L}_2 = \hat{M}_1 = x$ s in same segment $\hat{P}_3 = \hat{L}_2 = x$ alt s KP//MN $\hat{K} = \hat{P}_3$ $\therefore \Delta KLP$ is isosceles (Sides opp equal angles)	✓S/R ✓S✓R ✓S✓R ✓R	(6)
			[10]

QUESTION 7



	Construction: Mark off PA = FG and PB = FH and join AB. $\Delta PAB \equiv \Delta FGH$ Congruency s s $P\hat{A}B = \hat{G}$ $\hat{G} = \hat{Q}$ given $\therefore P\hat{A}B = \hat{Q}$ $\therefore AB//QR$ corresp. s equal $\frac{PQ}{PA} = \frac{PR}{PB}$ line // to one side of Δ $\therefore \frac{PQ}{FG} = \frac{PR}{FH}$	✓construction ✓S ✓R ✓S ✓R ✓S/R	**[6]**

QUESTION 8

8.1	$\hat{N}_2 = 30°$ ∠s opp = sides $\hat{N}_4 = 30°$ tan-chord theorem $\hat{M} = 30°$ corresponding ∠s NP//MO	✓S/R ✓S/R ✓S/R	(3)
8.2	$\hat{N}_2 + \hat{N}_3 = 90^O$ (∠in the semi-circle) $\therefore R\hat{N}T = 90° + 30°$ $= 120°$	✓S/R ✓CA	(2)
8.3.1	$In \Delta RNP and \Delta RTN$ $\hat{R} = \hat{R}$ common $\hat{N}_4 = \hat{T} = 30°$ proven $\hat{P}_2 = R\hat{N}T$ rem ∠ $\therefore \Delta RNP /// \Delta RTN$ (AAA)	✓S/R ✓S/R ✓R	(3)
8.3.2	$\frac{RN}{RT} = \frac{NP}{TN}$ (/// triangles) $RN.TN = RT.NP$ $\hat{W}_2 = 90° = T\hat{N}P$ corr∠; NP//MO $NW = WT$ line from centre ⊥ chord $\therefore NT = 2WT$ $\therefore 2WT.RN = RT.NP$ $TW.RN = \frac{1}{2}RT.NP$	✓S✓R ✓S/R ✓S/R ✓ substituting NT = 2WT	
			[13]

		TOTAL MARKS:	**100**