KWAZULU-NATAL PROVINCE

EDUCATION
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS

COMMON TEST

MARCH 2022

MARKS: 100

TIME: 2 hours

N.B. This question paper consists of 6 pages, 2 diagram sheets and an information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 8 questions.

2. Answer **ALL** questions.

3. Clearly show **ALL** calculations, diagrams, graphs, et cetera that you have used in determining your answers.

4. Answers only will not necessarily be awarded full marks.

5. An approved scientific calculator (non-programmable and non-graphical) may be used, unless stated otherwise.

6. If necessary, answers should be rounded off to TWO decimal places, unless stated otherwise.

7. Diagrams are NOT necessarily drawn to scale.

8. Number the answers correctly according to the numbering system used in this question paper. Write neatly and legibly.

QUESTION 1

Given the quadratic sequence: $5;\ x;\ y;\ 29;\ \ldots$ and its second constant difference is equal to 4.

1.1 Calculate the values of x and y. (4)

1.2 If x = 9 and y = 17, determine the n^{th} term of the quadratic sequence. (4)

1.3 Calculate the 50^{th} term of the sequence. (2)

[10]

QUESTION 2

The 2^{nd} term of an arithmetic sequence is 8 and the 7^{th} term is eleven times the value of the first term. Determine the first three terms of the sequence. **[7]**

QUESTION 3

3.1 Given:

$$\sum_{k=1}^{n} \frac{1}{p^{k-1}}$$

3.1.1 Write down the values of the first three terms of the series in terms of p. (1)

3.1.2 Determine the values of p for which the series is converging. (4)

3.2 The sum of the first n terms of a sequence is given by $4 - 4\left(\frac{1}{2}\right)^{n}$.

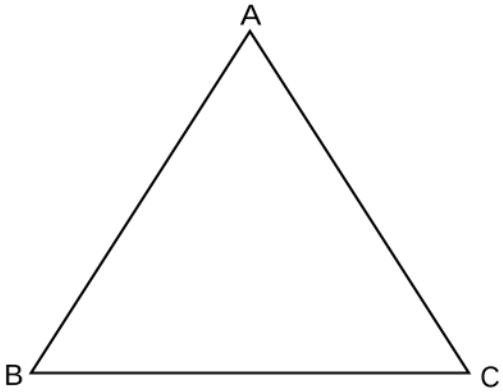
Calculate the first three terms. (5)

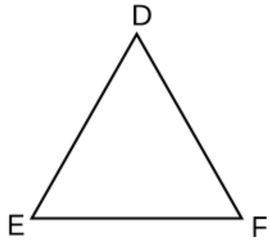
[10]

QUESTION 4

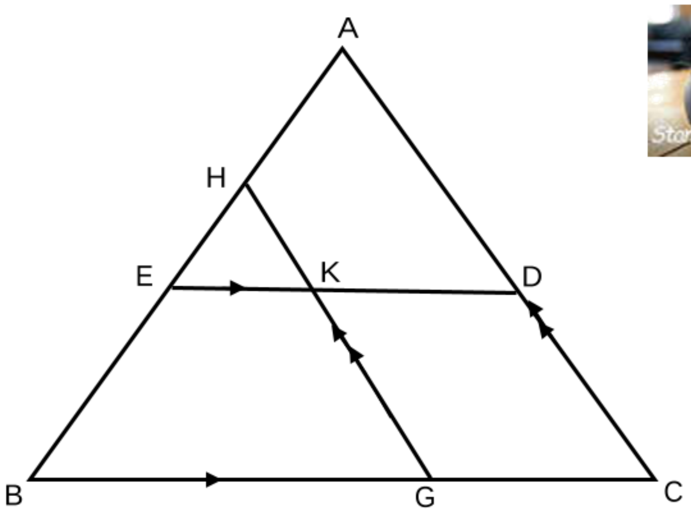
4.1 Given ΔABC and ΔDEF with $\hat{A} = \hat{D}, \hat{B} = \hat{E}$ and $\hat{C} = \hat{F}$.

Prove the theorem which states that $\frac{AB}{DE} = \frac{AC}{DF}$ (7)





4.2 In the figure, ΔABC has $HG \parallel AC$ and $ED \parallel BC$. ED and HG intersect at K.
$\frac{AD}{DC} = \frac{3}{2}$ and $BG = 2GC$. AB = 15 units.



Determine with reasons the value of:

4.2.1 AE (3)

4.2.2 AH (3)

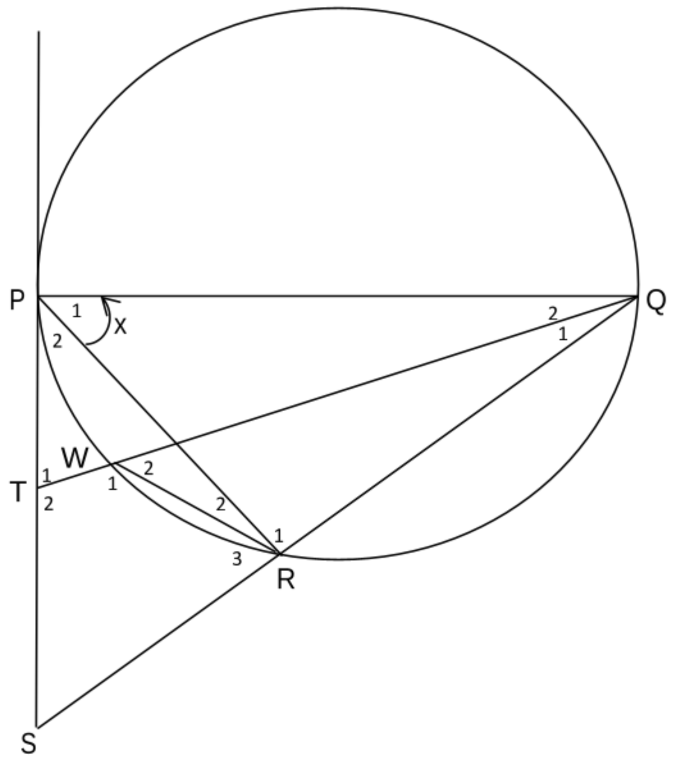
4.2.3 $\frac{GK}{KH}$ (3)

4.2.4 $\frac{\text{Area of } \Delta HEK}{\text{Area of } \Delta HBG}$ (3)

[19]

QUESTION 5

In the figure: PQ is the diameter of the circle. SP is the tangent to the circle at P.
QT intersects the circle at W and with T on the line SP. QS intersects the circle at R. $\hat{P}_1 = x$.



5.1	Give a reason why $P\hat{R}Q = 90^\circ$.	(1)
5.2	Prove $\hat{S} = x$.	(3)
5.3	Prove that SRWT is a cyclic quadrilateral.	(3)
5.4	Prove that $\Delta QWR \;|||\; \Delta QST$.	(3)
5.5	If QW = 5 cm, TW = 1 cm, QR = 4 cm and WR = 2 cm, calculate the lengths of:	
	5.5.1 TS	(3)
	5.5.2 SR	(3)
		[16]

QUESTION 6

6.1 Given $\cos 20^{\circ} = p$ and $\sin 14^{\circ} = q$

Without using a calculator, calculate the value of the following in terms of p, q or p and q.

6.1.1 $\sin 20^{\circ}$ (2)

6.1.2 $\cos 6^{\circ}$ (6)

6.2 Simplify into a single trigonometric ratio.

$$\sqrt{\frac{\frac{1}{2}\sin 2x}{\tan(540^{\circ}+x)\left(\frac{1}{\cos^2 x}-\tan^2 x\right)}}$$ (6)

[14]

QUESTION 7

7.1.1 Prove the following identity:

$$\cos 4x = 8\cos^4 x - 8\cos^2 x + 1$$ (4)

7.1.2 Hence, determine, without the use of a calculator, the general solution of

$$16\cos^4 x - 16\cos^2 x + 2 = 1$$ (5)

7.1.3 Write down the minimum value of the expression $16\cos^4 x - 16\cos^2 x + 2$. (2)

7.2 Calculate, without the use of a calculator, the value of:

$$\frac{2\sin^2 22.5^{\circ} - 1}{4\sin 22.5^{\circ}\cos 22.5^{\circ}}$$ (5)

[16]

QUESTION 8

8.1 Sketch the graphs of $f(x) = \sin(2x)$ and $g(x) = 2\cos x$ for the domain $x \in [-90^{\circ}; 180^{\circ}]$. (Use the axes provided) (6)

8.2 Use your graphs to determine the solution $\frac{g(x)}{f(x)} \geq 1$. (2)

[8]

Total: 100 marks

INFORMATION SHEET: MATHEMATICS
INLIGTING BLADSY

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $A = P(1-ni)$ $A = P(1-i)^n$ $A = P(1+i)^n$

$T_n = a + (n-1)d$ $S_n = \frac{n}{2}(2a + (n-1)d)$

$T_n = ar^{n-1}$ $S_n = \frac{a(r^n - 1)}{r-1}$; $r \neq 1$ $S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x[(1+i)^n - 1]}{i}$ $P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $y - y_1 = m(x - x_1)$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $m = \tan\theta$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $a^2 = b^2 + c^2 - 2bc.\cos A$ area $\Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$ $\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum f.x}{n}$ $\sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

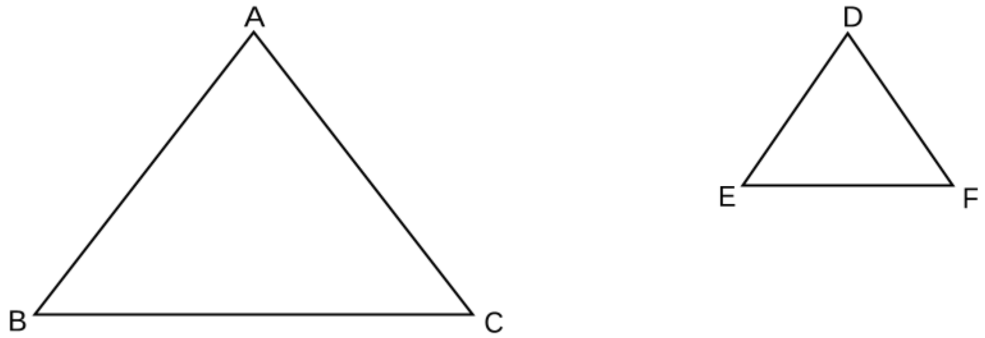
$\hat{y} = a + bx$ $b = \frac{\sum(x-\bar{x})(y-\bar{y})}{\sum(x-\bar{x})^2}$

NAME: ____ **GRADE:** __________

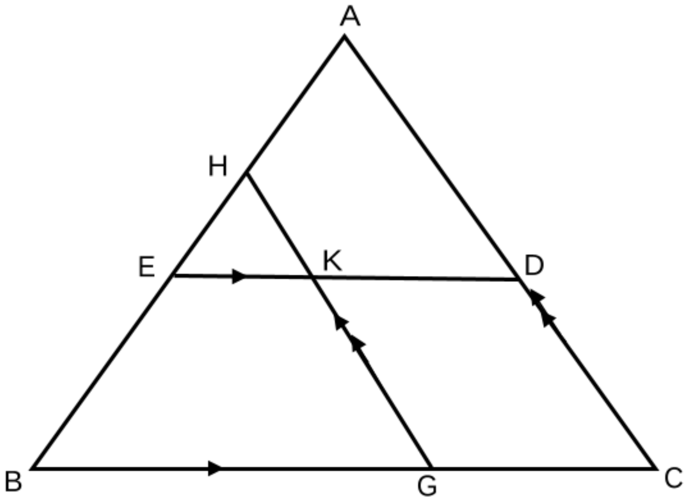
TEAR-OFF PAGE

DIAGRAM SHEET

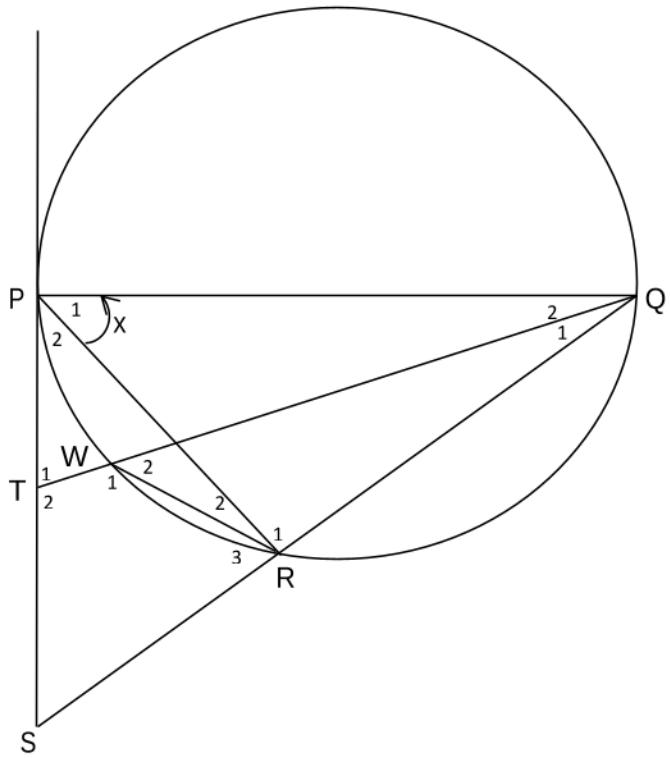
QUESTION 4.1



QUESTION 4.2



QUESTION 5



NAME: ____ **GRADE:** __________

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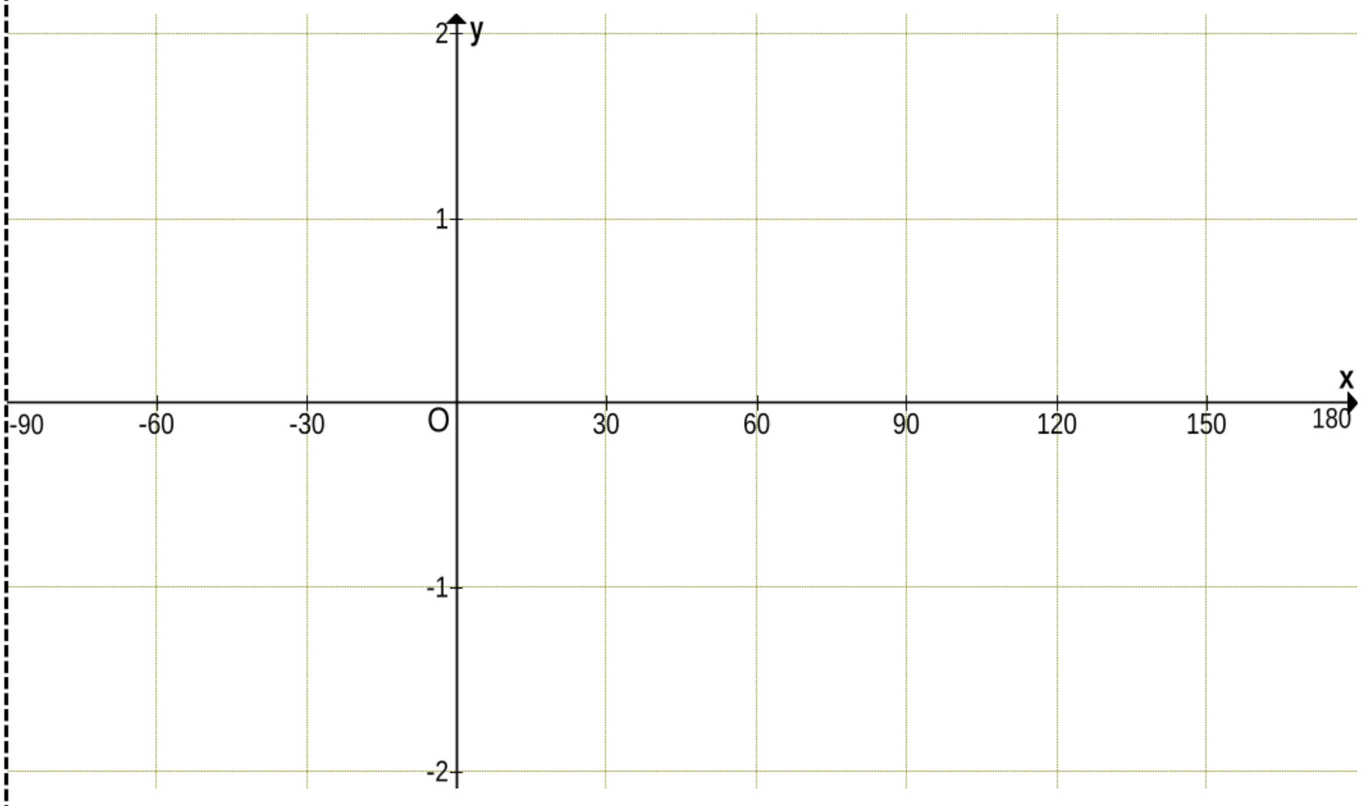
QUESTION 8



FINAL

KWAZULU-NATAL PROVINCE

EDUCATION
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS

MARKING GUIDELINE

COMMON TEST

MARCH 2022

MARKS: 100

This memorandum consists of 9 pages.

QUESTION 1

1.1	5 x y 29 1st Diff: $x-5$ $y-x$ $29-y$ 2nd Diff: $y-2x+5$ $29-2y+x$ $y-2x+5=4$ $y-2x=-1\ \ldots\ (1)$ $29-2y+x=4$ $-2y+x=-25\ \ldots\ (2)$ $(1)\times 2$: $2y-4x=-2\ \ldots\ (3)$ $(3)+(2)$: $-3x=-27 \quad \therefore\ x=9$ $\therefore\ y=17$	A✓setting up equation 1st line A✓setting up equation 1st line CA✓x – value CA✓ y – value	(4)
1.2	$2a=4 \quad \therefore a=2$ $3a+b=4 \quad \therefore b=-2$ $a+b+c=5 \quad \therefore c=5$ $T_n=2n^2-2n+5$ **OR** $2a=4 \quad \therefore a=2$ $3a+b=5 \quad \therefore b=-2$ $\therefore c=T_0=5$ $T_n=2n^2-2n+5$ **OR** $T_n=T_1+(n-1)d_1+\frac{(n-1)(n-2)}{2}d_2$ **OR** $T_n=\frac{(n-1)}{2}[2a+(n-2)d]+T_1$	A✓ a – value CA✓b – value CA✓c – value CA✓n^{th} term **OR** A✓a – value CA✓b – value CA✓c – value CA✓n^{th} term **OR** **OR**	(4) (4)
1.3	$T_{50}=2(50)^2-2(50)+5=4905$	CA✓substitution (from 1.2) CA✓answer	(2)
			[10]

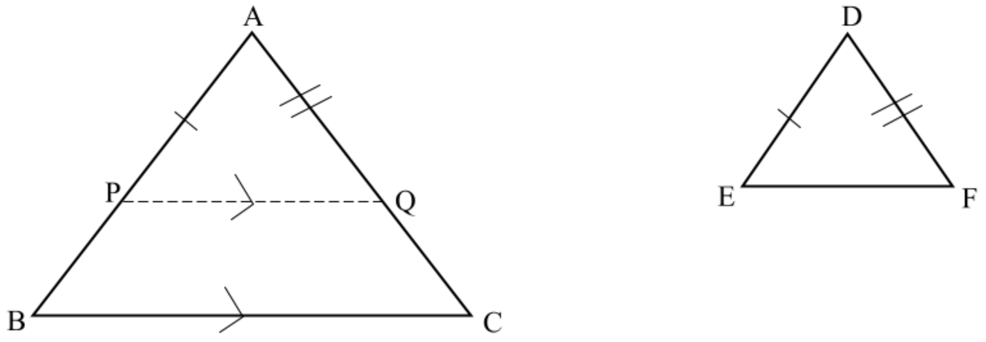
QUESTION 2

2	$a+d=8 \quad \rightarrow (1)$ $a+6d=11a \quad \rightarrow (2)$ $d=8-a \quad \rightarrow (3)$ $a+6(8-a)=11a$ $a+48-6a=11a$ $48=16a$ $a=3$ $d=5$ $3\,;8\;;\;13\;;\;...$	A✓equation (1) A✓equation (2) CA✓making *d/a* the subject CA✓correct substitution of *d/a* CA✓*a/d* value CA✓*d/a* value CA✓sequence	[7]

QUESTION 3

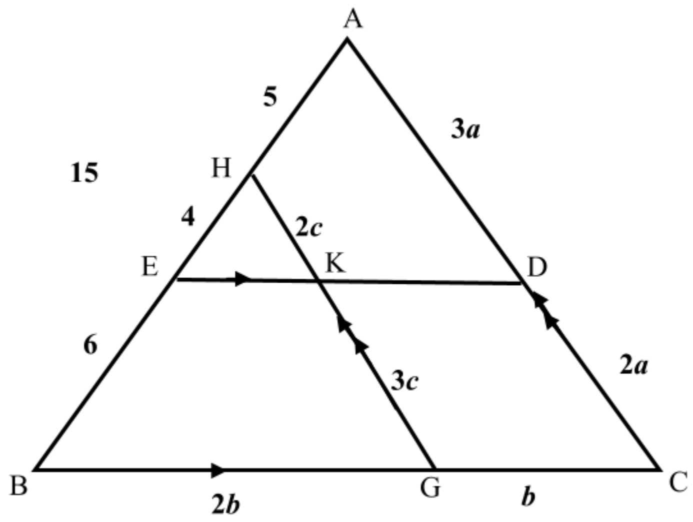
3.1.1	$1\,;\frac{1}{p}\,;\frac{1}{p^2}$	A✓All three terms	(1)
3.1.2	$-1<r<1$ $-1<\frac{1}{p}<1$ $p<-1 \;\; or \;\; p>1$	A✓condition for convergence CA✓*r* value in terms of *p* CACA✓✓answers	(4)
3.2	$S_n=4-4\left(\frac{1}{2}\right)^n$ $T_1=S_1=4-4\left(\frac{1}{2}\right)^1=2$ $T_1+T_2=S_2=4-4\left(\frac{1}{2}\right)^2=3$ $T_1+T_2+T_3=S_3=4-4\left(\frac{1}{2}\right)^3=3\frac{1}{2}$ $\therefore T_2=1$ $\therefore T_3=\frac{1}{2}$	 A✓ first term value A✓ Sum of first two terms A✓Sum of first 3 terms CA✓second term value CA✓third term value	(5)
			[10]

QUESTION 4



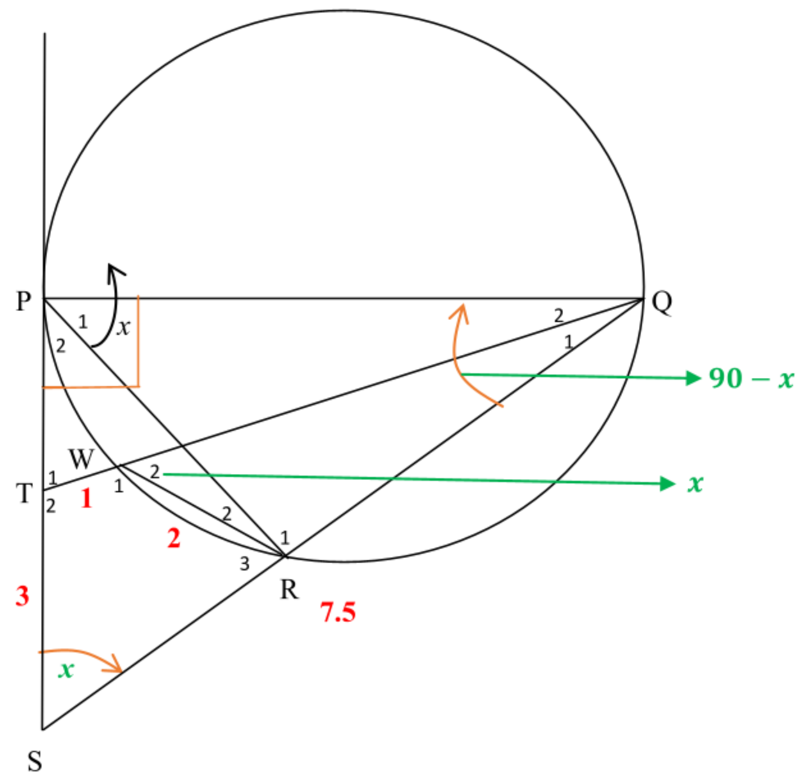
4.1	On AB mark off a point P such that AP = DE and on AC mark off a point Q such that AQ = DF. Join PQ. In $\Delta \mathbf{ABC}$ **and** $\Delta \mathbf{DEF}$ 1. AP = DE (Construction) 2. AQ = DF (Construction) 3. $\hat{\mathbf{A}} = \hat{\mathbf{D}}$ (Given) $\therefore \Delta \mathbf{APQ} \equiv \Delta \mathbf{DEF}$ (SAS) Now $\mathbf{A\hat{P}Q} = \mathbf{D\hat{E}F}$ But $\mathbf{D\hat{E}F} = \hat{\mathbf{B}}$ (Given) $\therefore \mathbf{A\hat{P}Q} = \hat{\mathbf{B}}$ PQ ǁ BC (Corresponding angles =) $\frac{\mathbf{AB}}{\mathbf{AP}} = \frac{\mathbf{AC}}{\mathbf{AQ}}$ (Prop. Thm. PQǁBC) $\frac{\mathbf{AB}}{\mathbf{DE}} = \frac{\mathbf{AC}}{\mathbf{DF}}$ (Construction AP = DE and AQ = DF)	✓S Construction ✓S/R ✓S ✓S ✓S/R ✓S/R ✓R	[7]

4.2



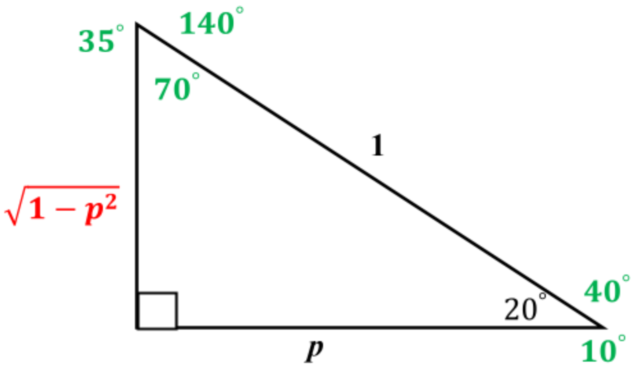
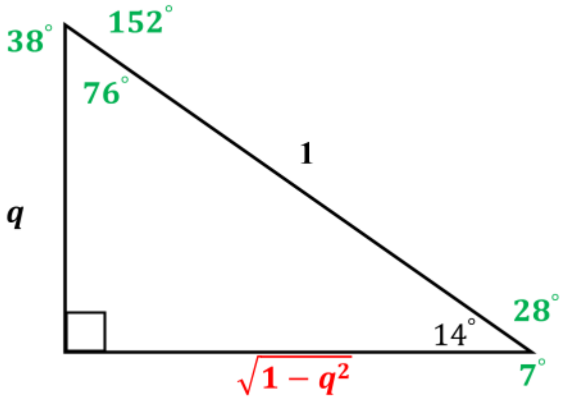
4.2.1	$\frac{AE}{15}=\frac{AD}{AC}=\frac{3}{5}$ … (Prop. Thm. ED ∥ BC) $AE=9$ units	✓S✓R ✓Answer	(3)
4.2.2	$\frac{AH}{15}=\frac{CG}{CB}=\frac{1}{3}$ … (Prop. Thm. HG ∥ AC) $AH=5$ units	✓S✓R ✓Answer	(3)
4.2.3	HE = 4 units and BH = 6 units $\frac{GK}{KH}=\frac{BE}{EH}=\frac{6}{4}=\frac{3}{2}$ … (Prop. Thm. EK ∥ BG)	CA✓HE = 4 units CA✓ S A✓ R	(3)
4.2.4	$\frac{\Delta HEK}{\Delta HEG}=\frac{2}{5}$ … (Δs with equal altitudes) $\frac{\Delta HEG}{\Delta HBG}=\frac{4}{10}=\frac{2}{5}$ … (Δs with equal altitudes) $\frac{\text{Area of }\Delta HEK}{\text{Area of }\Delta HBG}=\frac{\Delta HEK}{\Delta HEG}\times\frac{\Delta HEG}{\Delta HBG}=\frac{2}{5}\times\frac{2}{5}=\frac{4}{25}$ **OR** $\frac{\Delta HEK}{\Delta HBG}=\frac{\frac{1}{2}.(4)(2c)\sin E\hat{H}K}{\frac{1}{2}.(10)(5c)\sin E\hat{H}K}$ $\frac{\Delta HEK}{\Delta HBG}=\frac{4}{25}$ **OR** $\Delta HEK=\frac{2}{5}(\Delta HEG)$ … (Δs with equal altitudes) $=\frac{2}{5}\left(\frac{2}{5}\Delta HBG\right)$ … (Δs with equal altitudes) $\frac{\text{Area of }\Delta HEK}{\text{Area of }\Delta HBG}=\frac{4}{25}$	✓S/R ✓S/R ✓Answer **OR** ✓Area of numerator ✓Area of Denominator ✓Answer **OR** ✓S/R ✓S/R ✓Answer	(3) (3) (3)
			[19]

QUESTION 5



5.1	Angle subtended by the diameter on a circle/ Angle in a semi-circle	✓ R	(1)
5.2	$\mathbf{P\hat{Q}R = 90^\circ - x}$ … (Sum of angles of Δ) $\mathbf{S\hat{P}Q = 90^\circ}$ … (Diameter perpendicular to tangent) $\mathbf{\hat{S} = x}$ … (Sum of angles of Δ)	✓S ✓S✓R	(3)
5.3	$\mathbf{Q\hat{W}R = x}$ … (Angles in the same segment are equal) $\mathbf{Q\hat{W}R = \hat{S} = x}$ SRWT is a cyclic quad. … (Converse of exterior angle of cyclic quad equal to interior opposite angle)	✓S/R ✓$\mathbf{Q\hat{W}R = \hat{S} = x}$ ✓R	(3)
5.4	In Δ's QWR and QST 1) $\mathbf{Q\hat{W}R = \hat{S} = x}$ …….. (Proven) 2) $\mathbf{\hat{Q}_1}$ is common 3) $\mathbf{\hat{T}_2 = W\hat{R}Q}$ …………… (Remaining angles of Δ's) ΔQWR ||| ΔQST …. (AAA)	✓S ✓S ✓R	(3)
5.5.1	$\mathbf{\frac{QW}{QS} = \frac{WR}{ST} = \frac{QR}{QT}}$…. (ΔQWR ||| ΔQST) $\mathbf{\frac{2}{ST} = \frac{4}{6}}$ ST = 3 cm 	✓S/R ✓substitution CA✓Answer	(3)
5.5.2	$\mathbf{\frac{5}{QS} = \frac{2}{3}}$ …. (ΔQWR ||| ΔQST) QS = 7,5 cm	CA✓ S ✓R CA✓Answer	(3)
			[16]

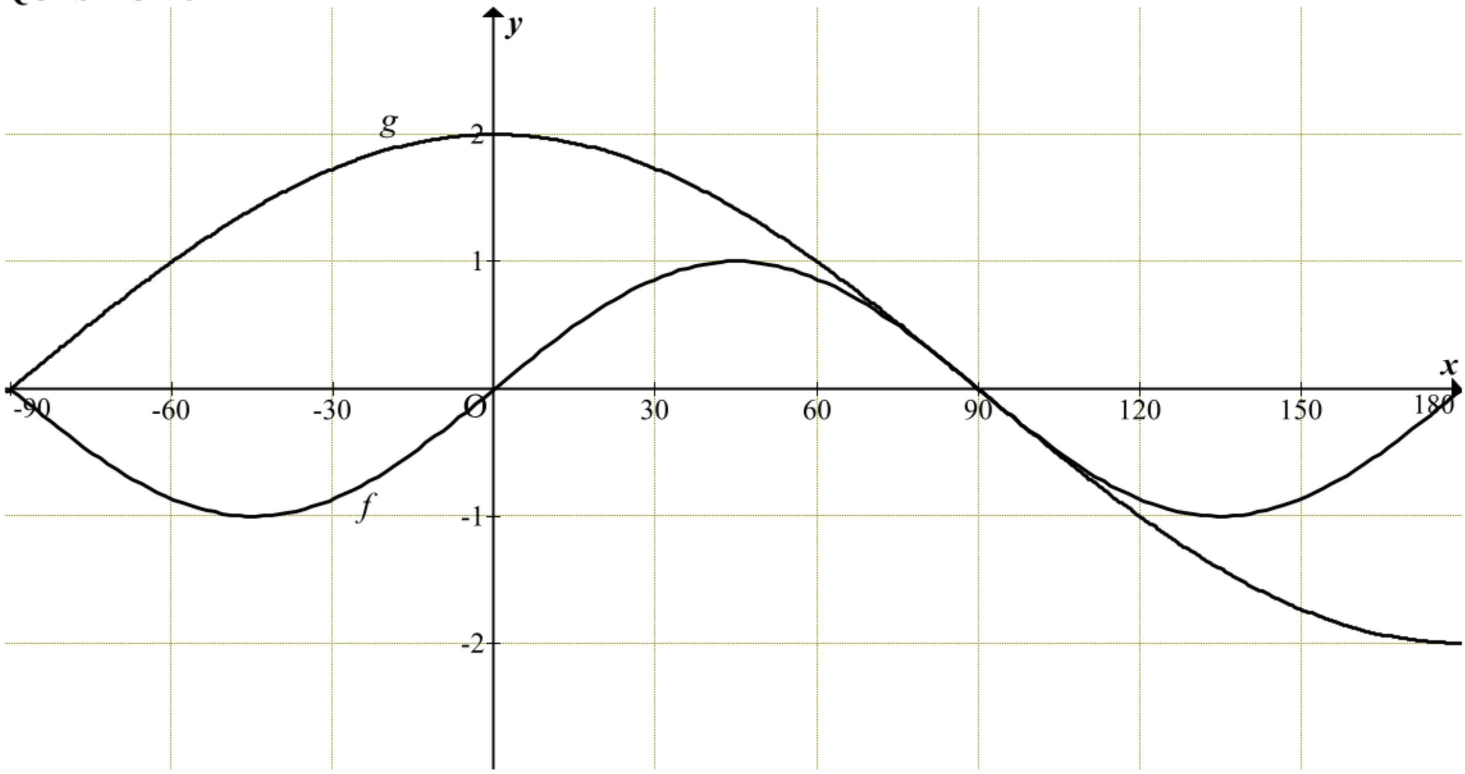
QUESTION 6

6.1.1	  $\sin 20^\circ = \sqrt{1-p^2}$	M✓ A✓ Answer	(2)
6.1.2	$\cos 6^\circ = \cos(20^\circ - 14^\circ)$ $= \cos 20 \cos 14 + \sin 20 \sin 14$ $= p.\sqrt{1-q^2} + \sqrt{1-p^2}.\text{q}$	A✓Writing as difference A✓Expansion A ✓ ($\cos 20^\circ$) value CA ✓$\sqrt{1-q^2}$CA✓$\sqrt{1-p^2}$ A ✓ ($\sin 14^\circ$) value	(6)
6.2	$\sqrt{\dfrac{\frac{1}{2}\sin 2x}{\tan(540^\circ + x)\left(\frac{1}{\cos^2 x} - \tan^2 x\right)}}$ $= \sqrt{\dfrac{\frac{1}{2}(2\sin x \cos x)}{\tan x.\left(\frac{1-\sin^2 x}{\cos^2 x}\right)}}$ $= \sqrt{\dfrac{\frac{1}{2}(2\sin x \cos x)}{\frac{\sin x}{\cos x}.\frac{\cos^2 x}{\cos^2 x}}}$ $= \sqrt{\sin x \cos x \times \dfrac{\cos x}{\sin x}}$ $= \sqrt{\cos^2 x}$ $= \cos x$	A✓expansion of $\sin 2x$ A✓reduction to $\tan x$ A✓$1 - \sin^2 x = \cos^2 x$ A✓$\tan x = \frac{\sin x}{\cos x}$ CA✓simplifying $\sqrt{\cos^2 x}$ CA✓answer	(6)
			[14]

QUESTION 7

7.1.1	$\cos 4x$ $= \cos(2.2x)$ $= 2\cos^2 2x - 1$ $= 2(2\cos^2 x - 1)^2 - 1$ $= 2(4\cos^4 x - 4\cos^2 x + 1) - 1$ $= 8\cos^4 x - 8\cos^2 x + 1$	A✓ $\cos 4x = \cos(2.2x)$ A✓expansion of $\cos 4x$ A✓ expansion of $\cos 2x$ A✓ squaring bracket	(4)
7.1.2	$2\cos 4x = 1$ $\cos 4x = \frac{1}{2}$ $4x = 60^\circ + 360k$ *or* $4x = 300^\circ/-60^\circ + 360k$ $x = 15^\circ + 90k$ *or* $x = 75^\circ/-15^\circ + 90k\, ; k\epsilon Z$	A✓setting up equation CA✓both solutions A✓$90k$ A✓ $k \in Z$ CA✓ $x = 15^\circ, 75^\circ/-15^\circ$	(5)
7.1.3	$16\cos^4 x - 16\cos^2 x + 2$ $= 2\cos 4x$ $\therefore$ Minimum Value $= -2$	A✓ A✓	(2)
7.2	$\frac{2\sin^2 22.5^\circ - 1}{4\sin 22.5^\circ \cos 22.5^\circ}$ $= \frac{-(1 - 2\sin^2 22.5^\circ)}{2.2\ \sin 22.5^\circ \cos 22.5^\circ}$ $= \frac{-\cos 45^\circ}{2.\sin 45^\circ}$ $= \frac{-1}{2} \times \frac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}}$ $= -\frac{1}{2} \times 1 = -\frac{1}{2}$ 	A✓re-writing numerator and denominator A✓Numerator A✓Denominator A✓ Special angle values/ co - ratios A✓Answer	(5)
			[16]

QUESTION 8



8.1	f: x-int., Maximum and Minimum and Shape g : y-int., Maximum and Minimum and Shape	AAA✓✓✓ AAA✓✓✓	(6)
8.2	$\frac{g(x)}{f(x)} \geq 1$ $g(x) \geq f(x); f(x) > 0$ $x \in (0^\circ; 180^\circ)\ ; x \neq 90^\circ$	AA✓✓Answer	(2)
			[8]

Total Marks: 100