

# GAUTENG DEPARTMENT OF EDUCATION



## JOHANNESBURG NORTH DISTRICT

### 2021 GRADE 12 CONTROL TEST

### MATHEMATICS TERM1

**MARKS : 100**  
**TIME : 2 hours**

This question paper consist of 17 pages

## INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of **9** questions.
2. Answer **ALL** the questions.
3. Clearly show **ALL** calculations, diagrams, graphs, etc. which was used in determining the answers.
4. Answers only will not necessarily be awarded full marks.
5. Use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. Where necessary, answers should be rounded off to **TWO** decimal places, unless stated otherwise.
7. Diagrams are **NOT** necessarily drawn to scale.
- 8. ANSWER Question 7 on Annexure 7.1 - 7.2.2**
- 9. ANSWER Question 8 on Annexure 8.1 - 8.1.3**
- 10. ANSWER Question 9 on Annexure 9.1 - 9.2**
- 11. Tear off page 12 till page 17 . AND SUBMIT theses pages with your answer scripts .**
12. An **information sheet** is on page 11 of the question paper.
13. Number the questions correctly according to the numbering used in the question paper.
14. Write neatly and legibly.

**QUESTION 1**

1.1 Solve for  $x$ :

1.1.1  $(x - 5)(x + 1) = 0$  (2)

1.1.2  $2x^2 - 11x + 7 = 0$  (correct to two decimal places) (3)

1.1.3  $x - 5x^{\frac{1}{2}} = -6$  (4)

1.2 Calculate  $a$  and  $b$  if  $\sqrt{\frac{5^{2014} - 5^{2012}}{6}} = a(5^b)$  and  $a$  is not a multiple of 5. (4)

1.3 Solve for  $x$  and  $y$ :

$1 = 3y - x$  and  $y^2 + 2xy = 3x^2 - 7$  (7)

[20]

**QUESTION 2**

Given the arithmetic series:  $3 + 10 + 17 + \dots + 150$ .

2.1 Write down the fourth term in the series. (1)

2.2 Determine the general term of the series. (2)

2.3 Express the series in sigma notation. (1)

[4]

**QUESTION 3**

3.1 Consider the progression :  $3; \frac{1}{2}; 3; \frac{4}{10}; 3; \frac{16}{50}; \dots$

3.1.1 Write down the next TWO terms of the progression. (1)

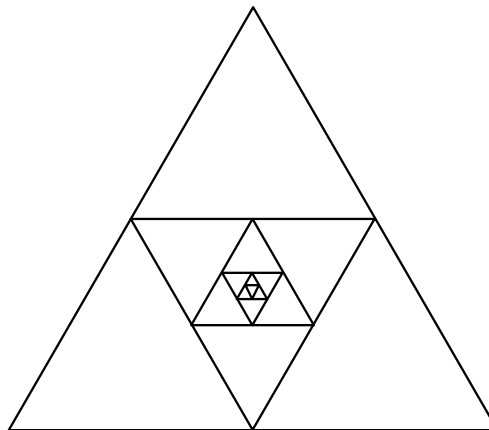
3.1.2 Calculate the sum of the first thirty-five terms of the progression. (5)

3.2 Calculate :  $\sum_{n=3}^{\infty} 5(3)^{1-n}$  (4)

[10]

**QUESTION 4**

In the diagram below, the 1<sup>st</sup> (outer) triangle is an equilateral triangle with sides of 8cm. A 2<sup>nd</sup> triangle is drawn within this triangle by joining the midpoints of the sides of the 1<sup>st</sup> triangle. This process is continued without end.



4.1 What is the perimeter of the 4<sup>th</sup> triangle? (2)

4.2 What is the perimeter of the  $n^{\text{th}}$  triangle? (3)

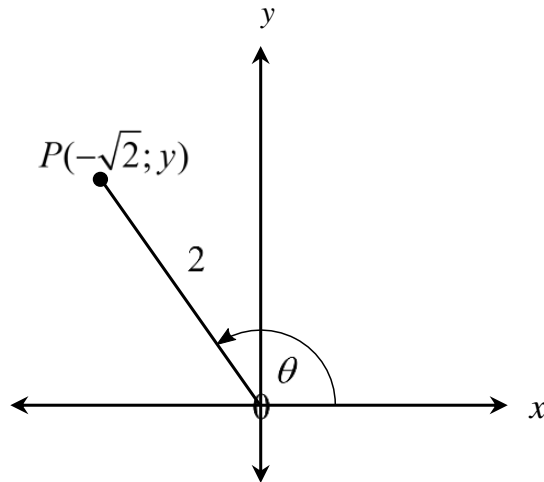
[5]



**QUESTION 5**

5.1 In the sketch below, P is a point on the Cartesian plane, with  $P\hat{O}X = \theta$ .

Use the sketch to determine the following:



5.1.1 The value of  $y$ . (2)

5.1.2 The value of  $\frac{2\sin\theta\cos\theta}{\cos^2\theta - 1}$  (5)

5.2 Simplify the following, WITHOUT USING A CALCULATOR:

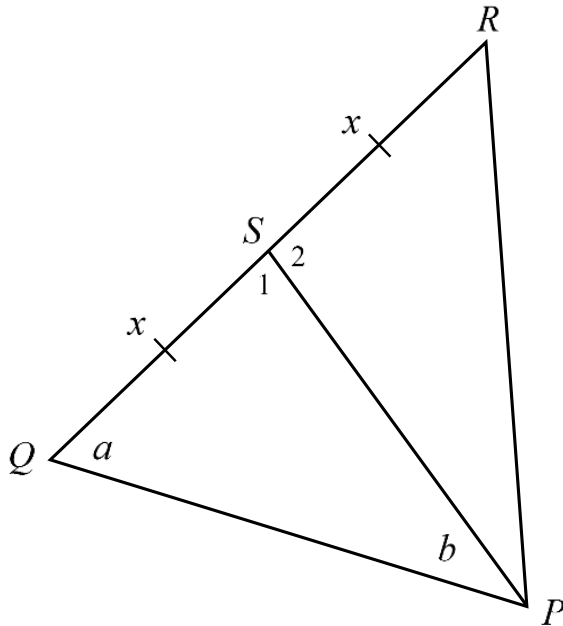
$$\frac{\cos(180^\circ + \theta) \cdot \tan(720^\circ - \theta) \cdot \sin^2(90^\circ - \theta)}{\sin(180^\circ - \theta)} + \sin^2\theta \quad (7)$$

5.3 If  $6\sin^2\theta - 4\cos^2\theta = -5\sin\theta \cdot \cos\theta$ , determine the general solution for  $\theta$ . (8)

[22]

**QUESTION 6**

In the sketch below, PS is the median of  $\triangle PQR$ , and thus  $QS = SR = x$ .  $\hat{Q} = a$  and  $\hat{QPS} = b$ .



6.1 Show that  $PS = \frac{x \sin a}{\sin b}$  (2)

6.2 Express the size of  $S_2$ , in terms of  $a$  and  $b$ , without reasons. (1)

6.3 Hence, show that: Area of  $\triangle PSR = \frac{x^2 \sin a \times \sin(a + b)}{2 \sin b}$  (3)

6.4 Determine the area of  $\triangle PSR$ , rounded to two decimal places, if  $x = 14,2\text{cm}$ ,  $a = 34^\circ$  and  $b = 41^\circ$ . (3)

[9]

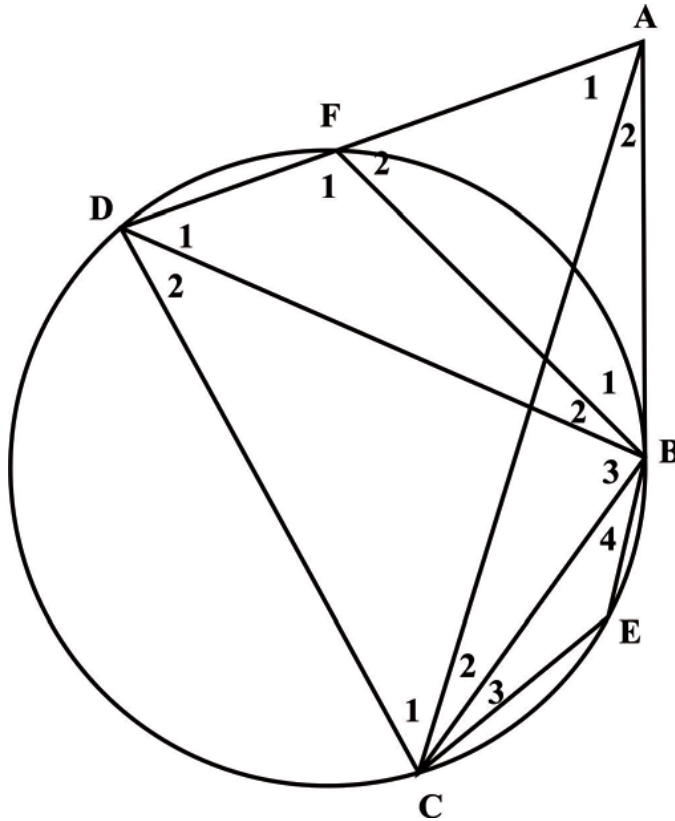
Give reasons for your statements and calculations in QUESTIONS 7, 8 and 9

Use the Annexure's provided to answer QUESTIONS 7, 8 and 9

**QUESTION 7**

7.1 In the diagram below, AB is a tangent to the circle passing through B, E, C and D

AD cuts the circle at F. AC is drawn.



Give reasons for the following statements:

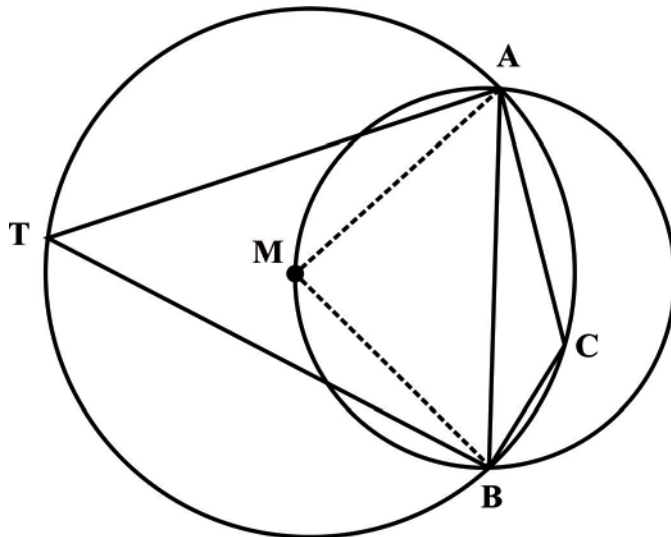
(5)

STATEMENT	REASONS
$\hat{C}_1 + \hat{C}_2 = \hat{F}_2$	
$\hat{D}_2 + \hat{E} = 180^\circ$	
$\hat{B}_1 = \hat{D}_1$	
$\hat{B}_2 + \hat{B}_3 + \hat{D}_1 + \hat{D}_2 = 180^\circ$	
$\hat{B}_2 + \hat{B}_1 = \hat{C}_1 + \hat{C}_2$	

7.2 In the diagram below, circle centre M intersects a second smaller circle at A and B.

A, C, B and T are points on circle M.

AB is the diameter of the smaller circle.



7.2.1 Determine the size of  $\hat{C}$ . (6)

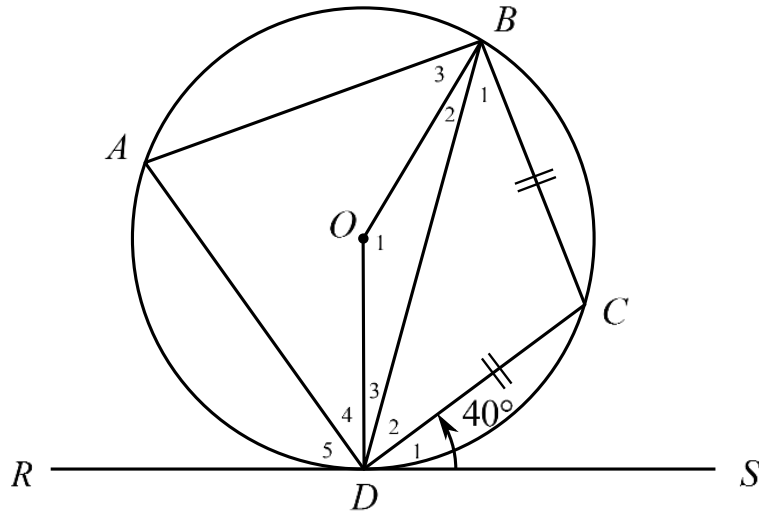
7.2.2 Explain why AMBC is not a cyclic quadrilateral. (1)

[12]

**QUESTION 8**

In the figure below, RDS is a tangent to circle O at D.  $BC = DC$ , and  $\hat{CDS} = 40^\circ$ .

Thus, calculate the size of the following angles, with reasons.



8.1  $\hat{BDC}$  (2)

8.2  $\hat{C}$  (2)

8.3  $\hat{A}$  (2)

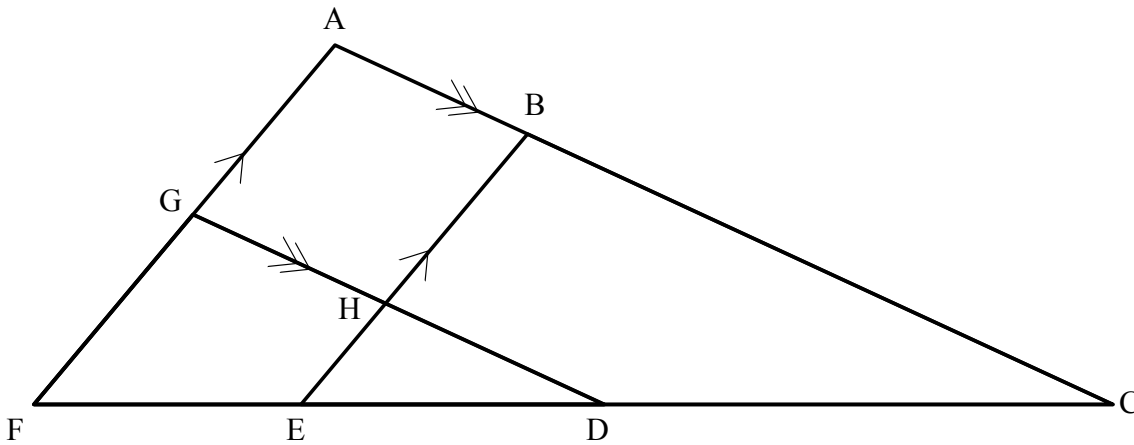
8.4  $\hat{O}_1$  (1)

[7]

**QUESTION 9**

The diagram below is the top view design of a new railway system. There are eight stations being built and these are labelled with letters from A- H. You have been asked to do some calculations for the railway company. As the engineer you know that:

- $AF \parallel BE$  and  $AC \parallel GD$ .
- $\frac{AB}{BC} = \frac{4}{7}$  and  $\frac{AG}{AF} = \frac{9}{17}$ .



9.1 Calculate

9.1.1  $\frac{FE}{FC}$ . (3)

9.1.2  $\frac{CD}{DF}$ . (2)

9.2 If the straight line distance of the track from F to C is 374 kilometres and it takes 50 hours to build one kilometre of the track, determine the number of hours it will take to build the section from E to D. (6)

[11]

**TOTAL 100 MARKS**

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni)$$

$$A = P(1 - ni)$$

$$A = P(1 - i)^n$$

$$A = P(1 + i)^n$$

$$\sum_{i=1}^n 1 = n$$

$$\sum_{i=1}^n i = \frac{n(n+1)}{2}$$

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1} ; r \neq 1$$

$$S_\infty = \frac{a}{1 - r} ; -1 < r < 1$$

$$F = \frac{x[(1+i)^n - 1]}{i}$$

$$P = \frac{x[1 - (1+i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

In  $\Delta ABC$ :

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$

$$\text{area} \Delta ABC = \frac{1}{2} ab \cdot \sin C$$

$$\sin(\alpha + \beta) = \sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cdot \cos \beta - \cos \alpha \cdot \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cdot \cos \beta - \sin \alpha \cdot \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cdot \cos \beta + \sin \alpha \cdot \sin \beta$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2 \sin^2 \alpha \\ 2 \cos^2 \alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2 \sin \alpha \cdot \cos \alpha$$

$$(x; y) \rightarrow (x \cos \theta - y \sin \theta ; y \cos \theta + x \sin \theta)$$

$$\bar{x} = \frac{\sum fx}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

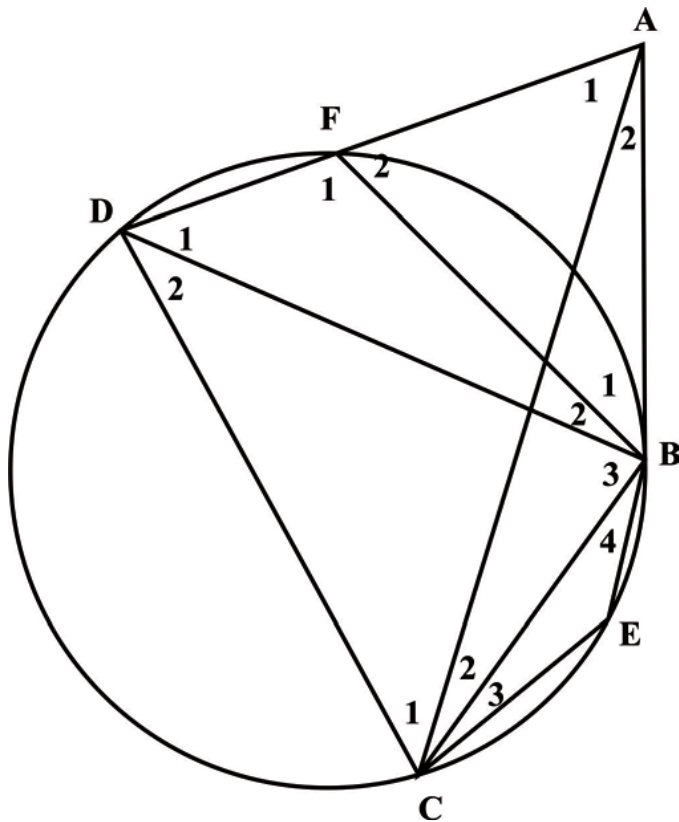
$$\hat{y} = a + bx$$

$$b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$$

**ANNEXURE 7.1 - 7.2 .2**

**QUESTION 7**

7.1



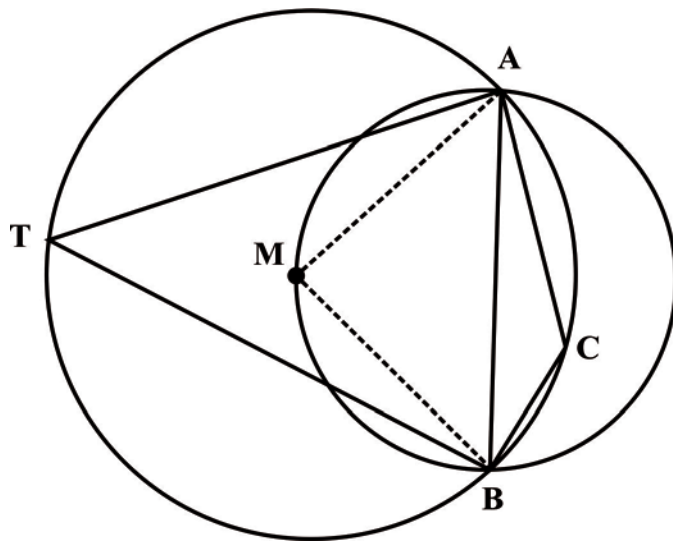
Give reasons for the following statements:

(5)

STATEMENT	REASONS
$\hat{C}_1 + \hat{C}_2 = \hat{F}_2$	
$\hat{D}_2 + \hat{E} = 180^\circ$	
$\hat{B}_1 = \hat{D}_1$	
$\hat{B}_2 + \hat{B}_3 + \hat{D}_1 + \hat{D}_2 = 180^\circ$	
$\hat{B}_2 + \hat{B}_1 = \hat{C}_1 + \hat{C}_2$	



7.2



7.2.1 Determine the size of  $\hat{C}$ .

(6)

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7.2.2 Explain why AMBC is not a cyclic quadrilateral.

(1)

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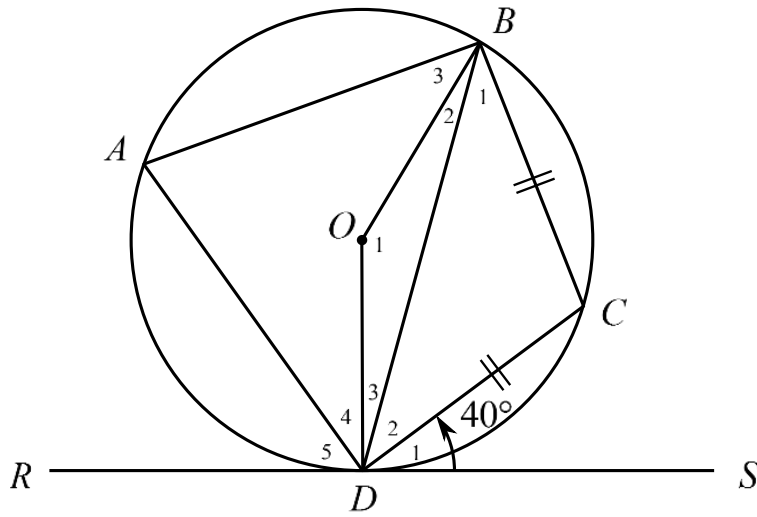
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[12]

**ANNEXURE 8.1 - 8.1.3**

**QUESTION 8**



8.1  $\hat{BDC}$  (2)

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8.2  $\hat{C}$  (2)

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8.3  $\hat{A}$

(2)

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8.4  $\hat{O}_1$

(1)

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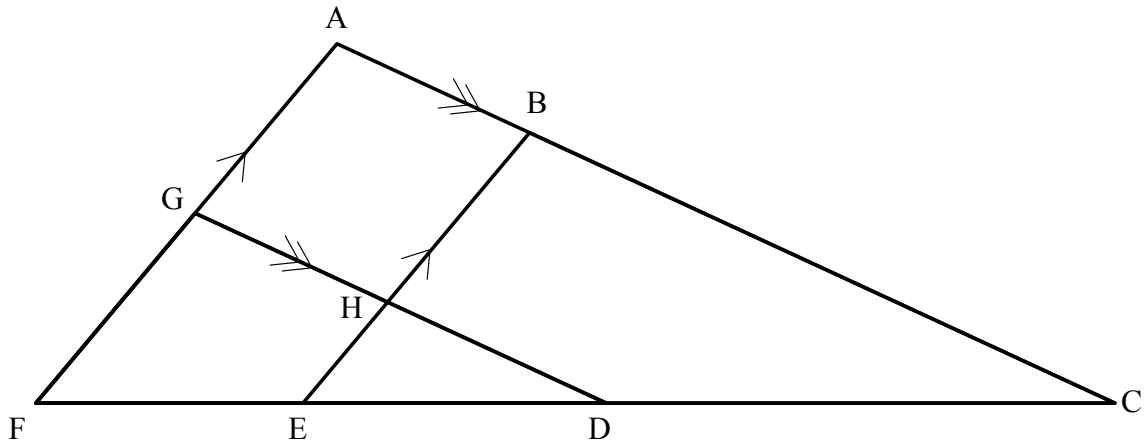
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[7]

**ANNEXURE 9.1 - 9.2**

**QUESTION 9**



9.1 Calculate

9.1.1  $\frac{FE}{FC}$ . (3)

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9.1.2  $\frac{CD}{DF}$ . (2)

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# GAUTENG DEPARTMENT OF EDUCATION



**JOHANNESBURG NORTH DISTRICT**

**2021  
GRADE 12  
CONTROL TEST**

**MATHEMATICS  
TERM 1**

## **MARKING GUIDELINES**

**MARKS : 100**  
**TIME : 2 hours**

<b>QUESTION 1</b>		
1.1.1	$(x - 5)(x + 1) = 0$ $x = 5$ or $x = -1$	✓ $x = 5$ ✓ $x = -1$ (2)
1.1.2	$2x^2 - 11x + 7 = 0$ $x = \frac{-(-11) \pm \sqrt{(-11)^2 - 4(2)(7)}}{2(2)}$ $x = 4,77$ or $x = 0,73$	✓ Sub ✓ $x = 4,77$ ✓ $x = 0,73$ (3)
1.1.3	$x - 5x^{\frac{1}{2}} = -6$ $x - 5x^{\frac{1}{2}} + 6 = 0$ Let $k^2 = x$ and $k = x^{\frac{1}{2}}$ $\therefore k^2 - 5k + 6 = 0$ $(k - 3)(k - 2) = 0$ $k = 3$ or $k = 2$ But: $x^{\frac{1}{2}} = 3$ or $x^{\frac{1}{2}} = 2$ $(x^{\frac{1}{2}})^2 = (3)^2$ or $(x^{\frac{1}{2}})^2 = (2)^2$ $x = 9$ or $x = 4$	✓ Standard form ✓ factors ✓ squaring ✓ $x$ values (4)
1.2	$\sqrt{\frac{5^{2014} - 5^{2012}}{6}}$ $= \sqrt{\frac{5^{2012} \cdot 5^2 - 5^{2012}}{6}}$ $= \sqrt{\frac{5^{2012}(25 - 1)}{6}}$ $= \sqrt{4 \cdot 5^{2012}}$ $= 2(5^{1006})$ $\therefore a = 2.$ and $b = 1006$	✓ $\sqrt{\frac{5^{2012} \cdot 5^2 - 5^{2012}}{6}}$ ✓ $\sqrt{4 \cdot 5^{2012}}$ ✓ $a = 2.$ ✓ $b = 1006$ (4)

1.3	$1 = 3y - x \dots\dots\dots(1)$ $y^2 + 2xy = 3x^2 - 7 \dots\dots(2)$ $x = 3y - 1 \dots\dots\dots(3)$ Sub (3) into (2) $y^2 + 2y(3y - 1) = 3(3y - 1)^2 - 7$ $y^2 + 6y^2 - 2y = 27y^2 - 18y + 3 - 7$ $0 = 20y^2 - 16y - 4$ $0 = 5y^2 - 4y - 1$ $0 = (y - 1)(5y + 1)$ $y = 1$ or $y = \frac{-1}{5}$ Sub y-values into (3) $x = 3(1) - 1$ or $x = 3(\frac{-1}{5}) - 1$ $x = 2$ or $x = \frac{-8}{5}$	✓ Equation 3 ✓ Sub ✓ Simplification ✓ Standard form ✓ Factors ✓ y-values ✓ x-values  (7)
<b>20 MARKS</b>		

<b>QUESTION 2</b>		
2.1	$T_n = 24$	✓ Ans  (1)
2.2	$T_n = 3 + (n - 1)(7)$ $T_n = 7n - 4$	✓ Sub ✓ Ans  (2)
2.3	$\sum_{k=1}^{22} (7k - 4)$	✓ Ans  (1)
<b>4 MARKS</b>		



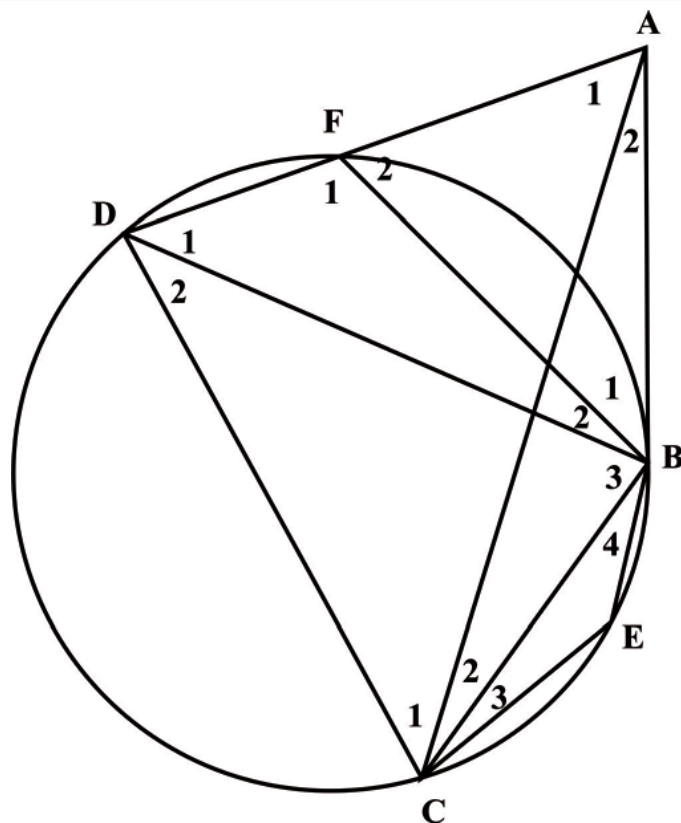
<b>QUESTION 3</b>		
3.1.1	3; $\frac{64}{250}$	✓ Ans (1)
3.1.2	$3 \times 18 = 54$ $S_{17} = \frac{\frac{1}{2}[(\frac{4}{5})^{17} - 1]}{(\frac{4}{5}) - 1}$  $= 2,44$  $\therefore S_{35} = 56,44$	✓ Odd terms ✓ Sub ✓ 2,44  ✓ ✓ $S_{35} = 56,44$ (5)
3.2	$T_1 = \frac{5}{9}$  $T_2 = \frac{5}{27}$  $\therefore r = \frac{1}{3}$  $S_{\infty} = \frac{\frac{5}{9}}{1 - \frac{1}{3}}$  $= \frac{5}{6}$ or 0,83	✓ $a$ ✓ $r$ ✓ Sub ✓ Ans (4)
<b>10 MARKS</b>		

<b>QUESTION 4</b>		
4.1	3cm	✓ ✓ Ans (2)
4.2	24; 12; 6; 3 $r = \frac{1}{2}$  $T_n = 24(\frac{1}{2})^{n-1}$	✓ .Sequence ✓ ratio ✓ Ans (3)
<b>5 MARKS</b>		

<b>QUESTION 5</b>		
5.1.1	$y^2 = (2)^2 - (-\sqrt{2})^2$ $y = \sqrt{2}$	✓ method ✓ Answer (2)
5.1.2	$\frac{2\left(\frac{\sqrt{2}}{2}\right)\left(-\frac{\sqrt{2}}{2}\right)}{\left(-\frac{\sqrt{2}}{2}\right)^2 - 1} = \frac{2\left(-\frac{1}{2}\right)}{\frac{1}{2} - 1} = 2$	✓✓✓ substitution ✓ sign ✓ answer (5)
5.2	$\frac{\cos(180^\circ + \theta) \cdot \tan(720^\circ - \theta) \cdot \sin^2(90^\circ - \theta)}{\sin(180^\circ - \theta)} + \sin^2 \theta$ $= \frac{-\cos \theta \times -\tan \theta \times \cos^2 \theta}{\sin \theta} + \sin^2 \theta$ $= \frac{\cos \theta \times \frac{\sin \theta}{\cos \theta} \times \cos^2 \theta}{\sin \theta} + \sin^2 \theta$ $= \cos^2 \theta + \sin^2 \theta = 1$	✓ $-\cos \theta$ ✓ $-\tan \theta$ ✓ $\cos^2 \theta$ ✓ $\sin \theta$  ✓ Identity: $\tan \theta$ ✓ $\cos^2 \theta + \sin^2 \theta$ ✓ =1 (7)
5.3	$6\sin^2 \theta - 5\sin \theta \cos \theta - 4\cos^2 \theta = 0$ $(2\sin \theta - \cos \theta)(3\sin \theta + 4\cos \theta) = 0$ $\therefore 2\sin \theta = \cos \theta$ $\therefore \tan \theta = \frac{1}{2}$ Ref $\angle = 26,57^\circ$ $\therefore \theta = 26,57^\circ + n180^\circ$ OR: $\theta = 180^\circ + 26,57^\circ + n180^\circ$ $\therefore \theta = 206,57^\circ + n180^\circ$ AND: $3\sin \theta = -4\cos \theta$ $\therefore \tan \theta = -\frac{4}{3}$ Ref $\angle = 53,13^\circ$ $\therefore \theta = 180^\circ - 53,13^\circ + n180^\circ$ $\theta = 126,87^\circ + n180$ OR: $\theta = 360^\circ - 53,13^\circ + n180^\circ$ $\therefore \theta = 306,87^\circ + n180^\circ$	✓ =0 ✓ factors  ✓ $\tan \theta$  ✓ solution  ✓ solution  ✓ $\tan \theta$ ✓ solution  ✓ solution (8)
<b>22 MARKS</b>		

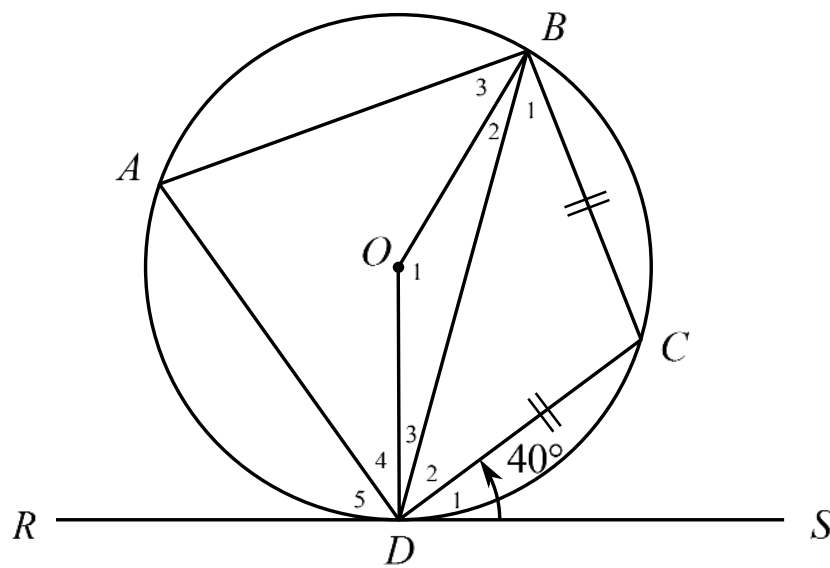
<b>QUESTION 6</b>		
6.1	$\frac{PS}{\sin a} = \frac{x}{\sin b}$ $\therefore PS = \frac{x \sin a}{\sin b}$	✓✓ method sine rule  (2)
6.2	$S_2 = a + b$	✓ accuracy  (1)
6.3	$Area = \frac{1}{2} PS \times SR \sin S_2$ $= \frac{1}{2} \left( \frac{x \sin a}{\sin b} \right) (x) (\sin(a + b))$ $= \frac{x^2 \sin a \cdot \sin(a + b)}{2 \sin b}$	✓ method: Area rule  ✓✓ sub in  (3)
6.4	$Area = \frac{(14,2)^2 \times \sin(34^\circ) \times \sin(34^\circ + 41^\circ)}{2 \sin(41^\circ)}$ $= 83,01 \text{ cm}^2$	✓✓ sub into formula  ✓ answer  (3)
<b>9 MARKS</b>		

**QUESTION 7**



7.1	STATEMENT	REASONS
	$\hat{C}_1 + \hat{C}_2 = \hat{F}_2$	✓ [ ext $\angle$ ' of cyclic quad ]
	$\hat{D}_2 + \hat{E} = 180^0$	✓ [ opp $\angle$ ' of cyclic quad ]
	$\hat{B}_1 = \hat{D}_1$	✓ [ tan chord ]
	$\hat{B}_2 + \hat{B}_3 + \hat{D}_1 + \hat{D}_2 = 180^0$	✓ [ opp $\angle$ ' of cyclic quad ]
	$\hat{B}_2 + \hat{B}_1 = \hat{C}_1 + \hat{C}_2$	✓ [ tan chord ]
7.2.1	$\hat{A}\hat{M}\hat{B} = 90^\circ$ ( $\angle$ 's in semi circle )  $\hat{T} = 45^\circ$ ( $\angle$ ' at center = $2 \times \angle$ at circum)  $\hat{C} = 135^\circ$ (opp $\angle$ 's of cyclic quad = 180)	✓ S ✓ R ✓ S ✓ R ✓ S ✓ R
7.2.2	$\hat{M} + \hat{C} \neq 180^\circ$ ; opp $\angle$ 's do not add up to 180°	✓ S & R
<b>12 MARKS</b>		

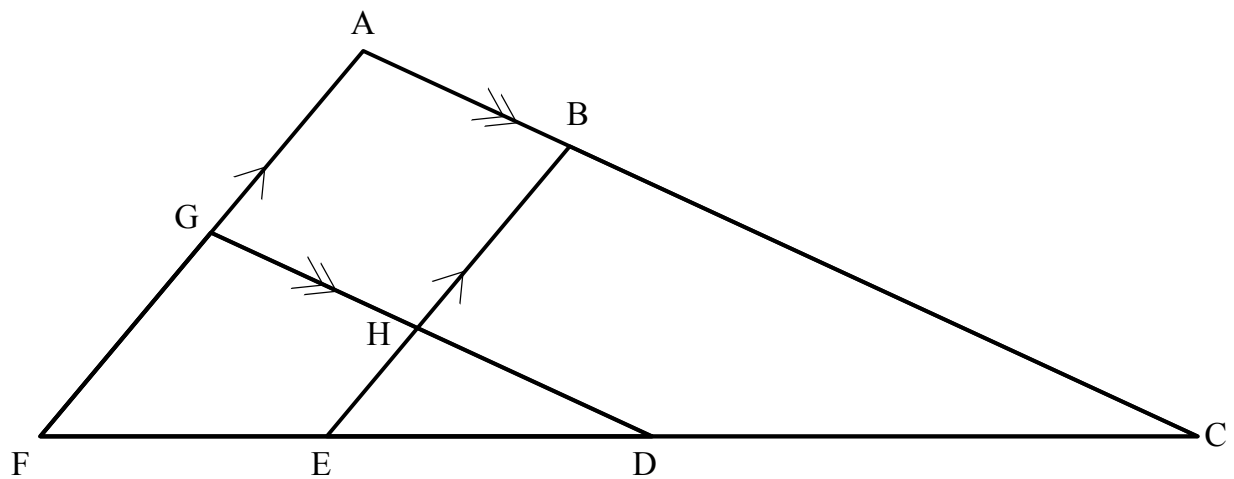
**QUESTION 8**



8.1	$B_1 = 40^\circ$ (tan-chord theorem) $\therefore BDC = 40^\circ$ ( $\angle$ 's opp = sides)	✓ S & R ✓ S & R	(2)
8.2	$C = 100^\circ$ ( $\angle$ 's in $\triangle$ )	✓ S ✓ R	(2)
8.3	$\therefore A = 80^\circ$ (opp $\angle$ 's in a cyclic quad)	✓ S ✓ R	(2)
8.4	$O_1 = 160^\circ$ ( $\angle$ at centre = $2\angle$ at circum)	✓ S & R	(1)

**7 MARKS**

**QUESTION 9**

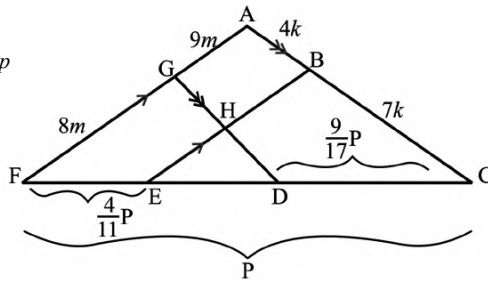


9.1.1	<p>Let <math>AB = 4k</math> and <math>BC = 7k</math></p> <p><math>\therefore \frac{FE}{FC} = \frac{AB}{AC} = \frac{4}{11}</math>; (Proportionality theorem OR using theorem on diagram)</p>	<p>✓ S ✓ S ✓ R</p> <p>(3)</p>
9.1.2	<p>Let <math>AG = 9m</math> and <math>AF = 17m</math></p> <p><math>\frac{CD}{DF} = \frac{AG}{GF} = \frac{9}{8}</math></p>	<p>✓ Reason</p> <p>✓ <math>\frac{CD}{DF} = \frac{AG}{GF} = \frac{9}{8}</math></p> <p>(2)</p>

9.2

If  $FC = p$  then  $ED = p - \frac{9}{17}p - \frac{4}{11}p$

$ED = \frac{20}{187}p$



The length of ED in kilometres is  $\frac{20}{187} \times 374 \text{ km} = 40 \text{ kilometres}$ .

It will take 2 000 hours to build the track from E to D.

**OR**

**Alternate:**

Let  $FE = 4p$  and  $EC = 7p$

$FD = 8m$  and  $DC = 9m$

$\therefore 11p = 374 \therefore p = 34$

$17m = 374 \therefore m = 22$

$\therefore DC = 374 - 4p - 9m$

$= 40 \text{ km}$

$\therefore 2\ 000 \text{ hours}$

**OR**

**Alternate:**

$FE = \frac{4}{11}(374) = 136$

$CD = \frac{9}{17}(374) = 198$

$\therefore ED = 374 - 136 - 198$

$= 40 \text{ km}$

$\therefore 4 \text{ hours} \rightarrow 40 \times 50$

$= 2\ 000 \text{ hours}$

✓ ✓

If  $FC = p$  then  $ED = p - \frac{9}{17}p - \frac{4}{11}p$

✓ ✓

$ED = \frac{20}{187}p$

✓

The length of ED in kilometres is  $\frac{20}{187} \times 374 \text{ km} = 40 \text{ kilometres}$ .

✓

It will take 2 000 hours to build the track from E to D.

**or**

✓

Let  $FE = 4p$  and  $EC = 7p$

✓

$FD = 8m$  and  $DC = 9m$

✓

$\therefore 11p = 374 \therefore p = 34$

✓

$17m = 374 \therefore m = 22$

✓

$\therefore DC = 374 - 4p - 9m = 40 \text{ km}$

✓

$\therefore 2\ 000 \text{ hours}$

**or**

$FE = \frac{4}{11}(374) = 136$

✓

✓

$CD = \frac{9}{17}(374) = 198$

✓ ✓

✓

$\therefore ED = 374 - 136 - 198 = 40 \text{ km}$

✓ ✓

$\therefore 4 \text{ hours} \rightarrow 40 \times 50 = 2\ 000 \text{ hours}$

(6)

11 MARKS

TOTAL MARKS 100