education

Department of Education
FREE STATE PROVINCE

GRADE 12

MATHEMATICS

MARCH TEST
2022

MARKS: 100

TIME: 2 hours

This question paper consists of 7 pages, 3 diagram sheets and 1 information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 7 questions.
2. Clearly show ALL calculations, diagrams, graphs, etc. that you have used to determine your answers.
3. Answers only will NOT necessarily be awarded full marks.
4. If necessary, round off answers to TWO decimal places, unless stated otherwise.
5. Diagrams are NOT necessarily drawn to scale.
6. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
7. The answer sheets for question 5, 6 and 7 are included in the question paper.
8. An information sheet with formulae is included at the end of the question paper.
9. Write neatly and legibly.

QUESTION 1

1.1 Solve for x:

1.1.1 $x(x+6)=0$ (2)

1.1.2 $3x^2+8x=-2$ (correct tot TWO decimal places) (4)

1.1.3 $x^2-64\leq 0$ (3)

1.1.4 $\sqrt{x+5}+1=x$ (5)

1.2 Solve simultaneously for x and y in the following equations:

$6x+5xy-5y=8$ (6)

$x+y=2$

[20]

QUESTION 2

2.1 Consider the quadratic number pattern: -20 ; -9 ; 0 ; 7 ; ...

2.1.1 Determine the n^{th} term . (4)

2.1.2 Determine the position and the value of the term with the highest value. (3)

[7]

QUESTION 3

3.1 Given the following arithmetic sequence: 13 ; 8 ; 3 ; ...

3.1.1 Determine the value of the 50^{th} term. (3)

3.1.2 Calculate the sum of the first fifty terms. (2)

3.2 Prove that: $a+a+d+a+2d+\ldots(\text{to } n \text{ terms})=\frac{n}{2}[2a+(n-1)d]$ (4)

3.3 Consider the geometric series: $3+m+\frac{m^2}{3}+\frac{m^3}{9}+\ldots$

3.3.1 For which value(s) of m will the series converge? (3)

3.3.2 It is given that: $3+m+\frac{m^2}{3}+\frac{m^3}{9}+\ldots=\frac{27}{7}$

Calculate the value of m (3)

3.4 Determine the value of n if:

$$\sum_{r=1}^{n}5.2^{1-r}=\frac{630}{64}$$

(6)

[21]

QUESTION 4

DO NOT USE A CALCULATOR FOR THIS QUESTION.

4.1 Given: $\tan\theta = \frac{3}{4}$; where $\theta \in [0^\circ ; 90^\circ]$

With the use of a sketch and without the use of a calculator, calculate:

4.1.1 $\sin\theta$ (3)

4.1.2 $\cos^2(90^\circ - \theta) - 1$ (2)

4.1.3 $1 - \sin 2\theta$ (3)

4.2 Simplify completely:

$$\frac{\sin^2(90^\circ + \alpha) + \sin(180^\circ + \alpha)\sin(-\alpha)}{\sin 180^\circ - \tan 135^\circ}$$ (5)

4.3 Prove the following identity:

$\sin 2\theta + \cos(2\theta - 90^\circ) = 4\sin\theta\cos\theta$ (3)

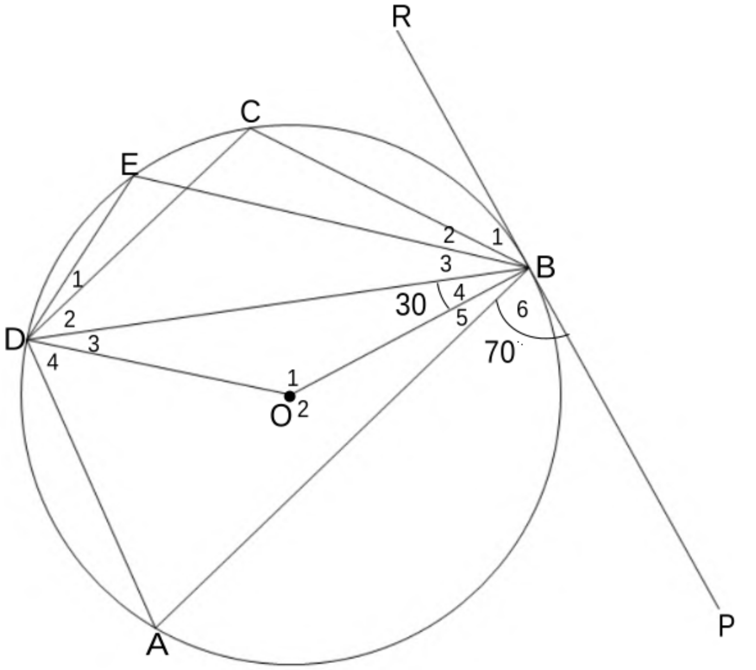
4.4 Solve for x if:

$20^{\sin x} + 20^{\sin x + 1} = 420$ for $-360^\circ \leq x \leq 360^\circ$ (5)

[21]

QUESTION 5

5.1 In the diagram below ABCD is a cyclic quadrilateral. RBP is a tangent to the circle with centre O. $B_4 = 30^\circ$ and $B_6 = 70^\circ$.



Determine with reasons the size of each of the following angles:

5.1.1 O_1 (2)

5.1.2 $\hat{A}$ (2)

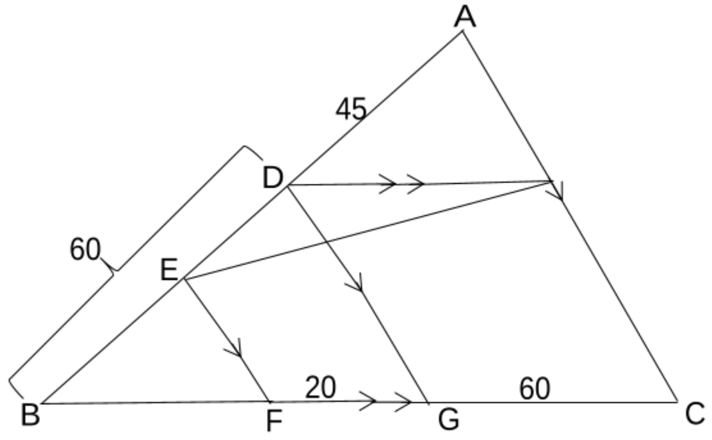
5.1.3 $\hat{C}$ (2)

5.1.4 ADB (2)

[8]

QUESTION 6

In the following diagram $AD = 45$, $BD = 60$, $GC = 60$ and $FG = 20$. $A\hat{B}C = 30^\circ$.



Determine the size of

6.1 BF (4)

6.2 DE (3)

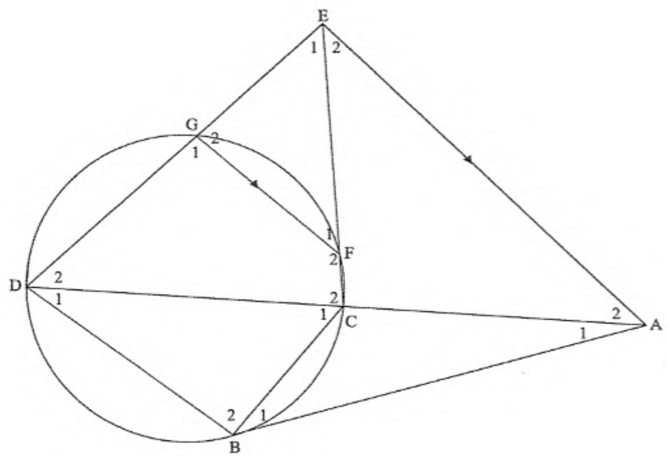
6.3 Calculate the area of $\triangle ABC$ (4)

[11]

QUESTION 7

In the diagram, DGFC is a cyclic quadrilateral and AB is a tangent to the circle at B.

Chords DB and BC are drawn. DG produced and CF produced meet in E and DC is produced to A. EA ∥ GF



7.1 Give a reason why $B_1=D_1$ (1)

7.2 Prove $\triangle ABC|||\triangle ADB$ (3)

7.3 Prove $E_2=D_2$ (4)

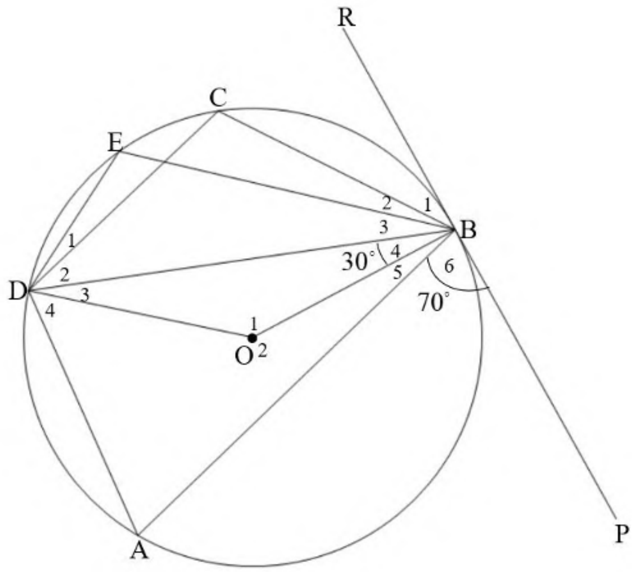
7.4 Prove $AE=\sqrt{AD\times AC}$ (4)

[12]

TOTAL: 100

QUESTION 5 **ANSWER SHEET**

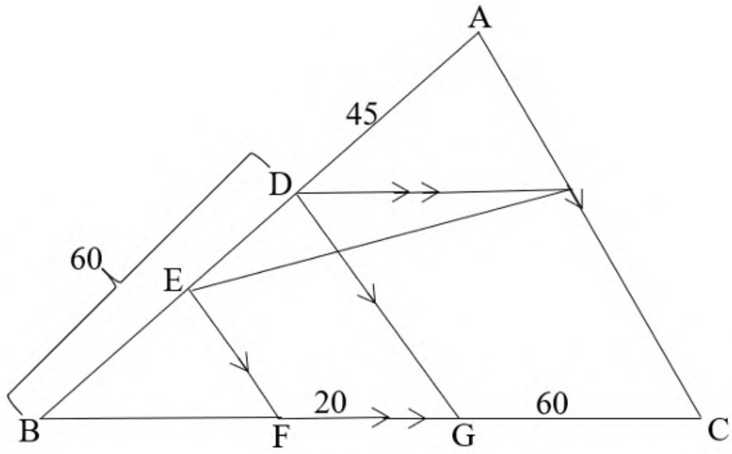
LEARNER NAME:……………………………… **GRADE 12:………….**



	STATEMENT	REASON	
5.1			
			(2)
5.2			
			(2)
5.3			
			(2)
5.4			
			(2)
			[8]

QUESTION 6 **ANSWER SHEET**

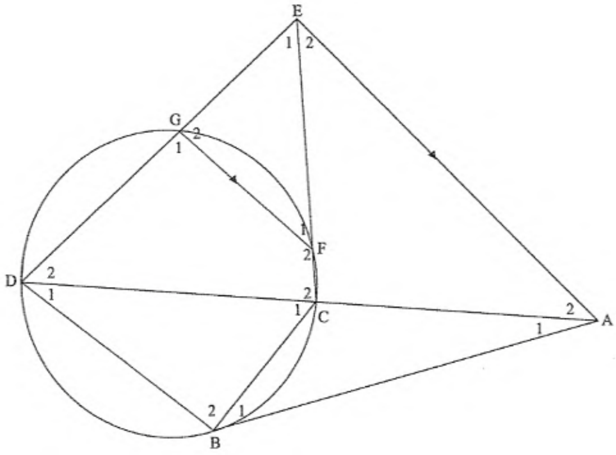
LEARNER NAME:……………………………… **GRADE 12:………….**



	STATEMENT	REASON	
6.1			
			(4)
6.2			
			(3)
6.3			
			(4)
			[11]

QUESTION 7 **ANSWER SHEET**

LEARNER NAME:……………………………… **GRADE 12:………….**



	STATEMENT	REASON	
7.1			
			(1)
7.2			
			(3)
7.3			
			(4)
7.4			
			(4)
			[12]

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\quad$ $A = P(1-ni)$ $\quad$ $A = P(1-i)^n$ $\quad$ $A = P(1+i)^n$

$T_n = a + (n-1)d$ $\quad$ $S_n = \frac{n}{2}[2a + (n-1)d]$

$T_n = ar^{n-1}$ $\quad$ $S_n = \frac{a(r^n - 1)}{r - 1}\ ;\ r \neq 1$ $\quad$ $S_\infty = \frac{a}{1-r}\ ;\ -1 < r < 1$

$F = \frac{x[(1+i)^n - 1]}{i}$ $\quad$ $P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad$ $M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad$ $y - y_1 = m(x - x_1)$ $\quad$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad$ $m = \tan\theta$

$(x-a)^2 + (y-b)^2 = r^2$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$a^2 = b^2 + c^2 - 2bc.\cos A$

$\text{area } \Delta ABC = \frac{1}{2} ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad$ $\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad$ $\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$

$\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum x}{n}$ $\quad$ $\sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad$ P(A or B) = P(A) + P(B) – P(A and B)

$\hat{y} = a + bx$ $\quad$ $b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$

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MARCH 2022

MARKING GUIDELINE

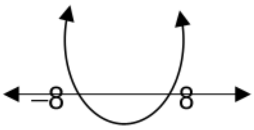
MARKS: 100

These marking guidelines consists of 9 pages.

NOTE:

- If a candidate answers a question TWICE, only mark the FIRST attempt.

QUESTION 1

1.1.1	$x(x+6)=0$ $x=0$ or $x=-6$	✓ $x=0$ ✓ $x=-6$ (2)
1.1.2	$3x^2+8x=-2$ $3x^2+8x+2=0$ $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ $x=\frac{-8\pm\sqrt{(8)^2-4(3)(2)}}{2(3)}$ $x=0,23$ or $x=-2,90$	 ✓standard form. ✓substitution into the correct formula. ✓ $x=0,23$ ✓ $x=-2,90$ (4)
1.1.3	$x^2-64\leq 0$ $(x+8)(x-8)\leq 0$ Critical Values: -8 and 8  $-8\leq x\leq 8$ OR $[-8;8]$	✓ factors ✓ diagram ✓ Answer (3)
1.1.4	$x\sqrt{x+5}+1=x$ $\sqrt{x+5}=x-1$ $\left(\sqrt{x+5}\right)^2=(x-1)^2$ $x+5=x^2-2x+1$ $x^2-3x-4=0$ $(x-4)(x+1)=0$ $x=4$ or $x=-1$ $\therefore x=4$ but $x\neq -1$	✓isolate $\sqrt{x+5}$ ✓squaring both sides ✓standard form ✓factors ✓conclusion (5)

1.2	$6x+5xy-5y=8$ and $x+y=2$ $x=2-y...(3)$ $6(2-y)+5(2-y)y-5y=8$ $12-6y+10y-5y^2-5y=8$ $5y^2+y-4=0$ $(5y-4)(y+1)=0$ $y=\frac{4}{5}$ or $y=-1$ $x=\frac{6}{5}$ or $x=3$	✓ $x-$ subject of the formula ✓substitution ✓standard form ✓factors ✓ $y-$ values ✓ $x-$ values (6)
		[20]

QUESTION 2

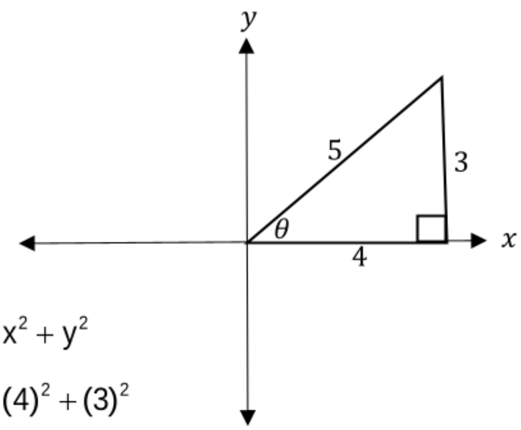
2.1.1	$-20;-9;0;7;...$ $11 \quad 9 \quad 7$ $-2 \quad -2$ $2a=-2$ $\qquad$ $3(-1)+b=11$ $-1+14+c=-20$ $a=-1$ $\qquad$ $b=14$ $c=-7$ $\therefore T_n=-n^2+14n-7$	✓value of a ✓ value of b ✓ value of c ✓ T_n (4)
2.1.2	$n=\frac{-b}{2a}$ $=\frac{-14}{2(-1)}$ $n=7$ $\therefore T_7=-(7)^2+14(7)-7$ $=42$	✓ $\frac{-14}{2(-1)}$ ✓value of n ✓Value of T_7 (3)
		[7]

QUESTION 3

3.1.1	$13; 8; 3; \ldots$ $a = 13$ and $d = -5$ $T_n = a + (n-1)d$ $T_{50} = 13 + (50-1) - 5$ $T_{50} = 57$	✓ $d = -5$ ✓substitution from the correct formula ✓Answer (3)
3.1.2	$S_n = \frac{n}{2}[2a + (n-1)d]$ $S_{50} = \frac{50}{2}[2(13) + (50-1)(-5)]$ $S_{50} = -5475$	✓Substitution from the correct formula ✓Answer (2)
3.2	$S_n = a + (a+d) + (a+2d) + \ldots + (l-2d) + (l-d) + l \ldots(1)$ $S_n = l + (l-d) + (l-2d) + \ldots(a+2d) + (a+d) + a \ldots(2)$ $2S_n = (a+l) + (a+1) + (a+l) + \ldots(a+l) + (a+l) + (a+l)$ $\therefore 2S_n = n(a+l)$ $\therefore S_n = \frac{n}{2}(a+l)$ $\therefore S_n = \frac{n}{2}[a + a + (n-1)d]$ $\therefore S_n = \frac{n}{2}[2a + (n-1)d]$	✓equation 1 and 2 ✓ $2S_n = n(a+l)$ ✓dividing by 2 ✓substitution of l (4)
3.3.1	$3 + m + \frac{m^2}{3} + \frac{m^3}{9} + \ldots$ $r = \frac{m}{3}$ $-1 < r < 1$ $-1 < \frac{m}{3} < 1$ $-3 < m < 3$	✓ $r = \frac{m}{3}$ ✓substitution of r ✓Answer (3)

3.3.2	$S_\infty = \frac{a}{1-r}$ $\frac{27}{7} = \frac{3}{1-\frac{m}{3}}$ $27 - \frac{27m}{3} = 21$ $27 - 9m = 21$ $6 = 9m$ $\therefore m = \frac{6}{9} = \frac{2}{3} = 0,67$	✓substitution ✓simplification ✓Answer (3)
3.4	$\sum_{r=1}^{n} 5.2^{1-r} = 5 + \frac{5}{2} + \frac{5}{4} + ...$ $S_n = \frac{a(1-r^n)}{1-r}$ $\frac{630}{64} = \frac{5\left[1-\left(\frac{1}{2}\right)^n\right]}{1-\frac{1}{2}}$ $\frac{63}{64} = 1 - \left(\frac{1}{2}\right)^n$ $\therefore \left(\frac{1}{2}\right)^n = \frac{1}{64}$ $\left(\frac{1}{2}\right)^n = \left(\frac{1}{2}\right)^6$ $n = 6$	✓ expansion to THREE terms ✓ $a = 2$ and $r = \frac{1}{2}$ ✓ subst into the correct formula ✓ simplification: $\frac{63}{64} = 1 - \left(\frac{1}{2}\right)^n$ ✓ same bases: $\left(\frac{1}{2}\right)^n = \left(\frac{1}{2}\right)^6$ ✓ answer (6)
		[21]

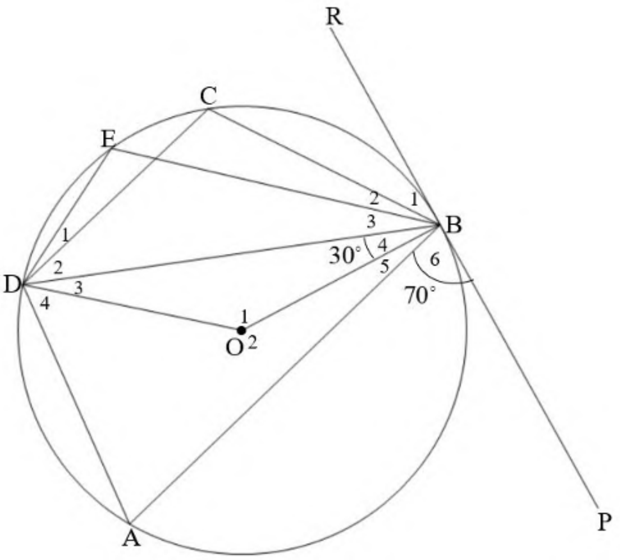
QUESTION 4

4.1.1	 $r^2 = x^2 + y^2$ $r^2 = (4)^2 + (3)^2$ $r = 5$ $\sin\theta = \frac{3}{5}$	✓diagram ✓ $r = 5$ ✓Answer (3)
4.1.2	$\cos^2(90^\circ - \theta) - 1$ $= \sin^2\theta - 1$ $= \left(\frac{3}{5}\right)^2 - 1$ $= \frac{-16}{25}$	✓ $\cos(90^\circ - \theta) = \sin\theta$ ✓Answer (2)
4.1.3	$1 - \sin 2\theta$ $= 1 - 2\sin\theta\cos\theta$ $= 1 - 2\left(\frac{3}{5}\right)\left(\frac{4}{5}\right)$ $= \frac{1}{25}$	✓double angle ✓substitution ✓Answer (3)
4.2	$\frac{\sin^2(90^\circ + \alpha) + \sin(180^\circ + \alpha)\sin(-\alpha)}{\sin 180^\circ - \tan 135^\circ}$ $= 4\sin\theta\cos\theta = \frac{\cos^2\alpha + (-\sin\alpha)(-\sin\alpha)}{0 - (-\tan 45^\circ)}$ $= \frac{\cos^2\alpha + \sin\alpha\sin\alpha}{0 + 1}$ $= \frac{\cos^2\alpha + \sin^2\alpha}{1}$ $= 1$	✓ $\cos^2\alpha$ ✓ $-\sin\alpha$ ✓ $\sin^2\alpha$ ✓ $\cos^2\alpha + \sin^2\alpha = 1$ ✓Answer (5)

4.3	$\sin 2\theta + \cos(2\theta - 90^\circ)$ $= \sin 2\theta + \sin 2\theta$ $= 2(2\sin\theta\cos\theta)$ $= 4\sin\theta\cos\theta$	✓ $\sin 2\theta$ ✓ $2\sin\theta\cos\theta$ ✓Answer (3)
4.4	$20^{\sin x} + 20^{\sin x+1} = 420$ for $-360^\circ \leq x \leq 360^\circ$ $\therefore 20^{\sin x}(1+20) = 420$ $\therefore 20^{\sin x} = 20$ $\therefore \sin x = 1$ $x = 90^\circ$ ref $\angle$ $x = -270^\circ$ or $x = 90^\circ$	✓split into a product of 2 bases ✓simplification / factorisation ✓dividing by 21 ✓equating exponents ✓both solutions (5)
		[21]

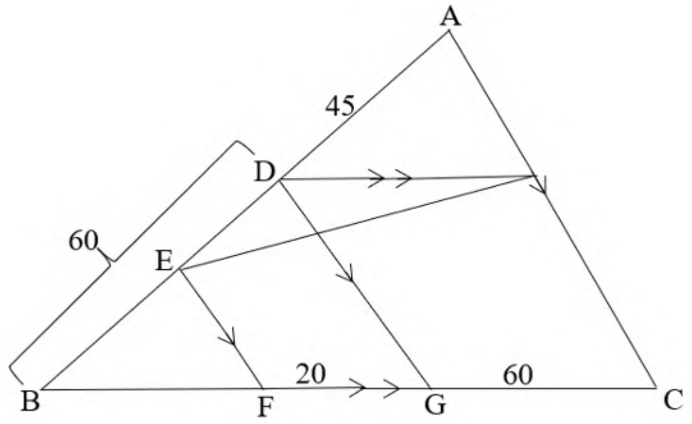
QUESTION 5

5.1



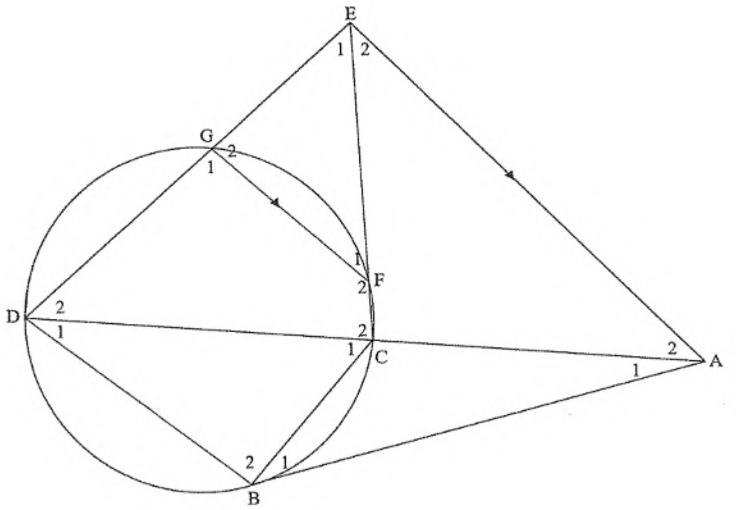
	STATEMENT	REASON	
5.1.1	$B_4 = D_2 = 30°$	$\angle's$ opp $=$ sides ($OB = D0$) radii	✓S and R
	$\therefore O_1 = 120°$	Sum of $\angle's$ of Δ	✓S and R (2)
5.1.2	$\hat{A} = 60°$	$\angle$ at centre $= 2 \times \angle$ at the circum.	✓$\hat{A} = 60°$ ✓R (2)
5.1.3	$C = 120°$	opp. $\angle's$ of a cyclic quad	✓ $C = 120°$ ✓R (2)
5.1.4	$ADB = 70°$	tan-chord theorem	✓ $ADB = 70°$ ✓R (2)
			[8]

QUESTION 6



	STATEMENT	REASON	
6.1	$\frac{BG}{60}=\frac{60}{45}$ $BG=80$ $\therefore BF=60$	Line $\parallel$ one side of Δ	✓S ✓R ✓ $BG=80$ ✓answer (4)
6.2	$\frac{ED}{60}=\frac{20}{80}$ $\therefore ED=15$	Line $\parallel$ one side of Δ	✓S ✓R ✓answer (3)
6.3	Area of $\Delta ABC=\frac{1}{2}AB.AC\sin B$ $=\frac{1}{2}(60+45)(60+20+60)\sin 30^\circ$ $=3675\text{units}^2$		✓ $AB=60+45$ ✓ $AC=60+20+60$ ✓substitution of Area ✓Answer (4)
			[11]

QUESTION 7

			
	STATEMENT	**REASON**	
7.1		tangent-chord theorem	✓Reason (1)
7.2	In △ ABC and △ ADB $\hat{A}_1 = \hat{A}_1$ $\hat{B}_1 = \hat{D}_1$ $\therefore \Delta ABC \| \| \| \Delta ADB$	 common proven ∠; ∠; ∠	 ✓S ✓ S ✓R (3)
7.3	$\hat{E}_2 = \hat{F}_1$ $\hat{F}_1 = \hat{D}_2$ $\therefore \hat{E}_2 = \hat{D}_2$	alternate ∠s; EA ∥ GF ext ∠ of cyc quad DGFC	✓S ✓R ✓S ✓R (4)
7.4	In △ AEC and △ ADE : $\hat{A}_2 = \hat{A}_2$ $\hat{E}_2 = \hat{D}_2$ $\therefore \triangle AEC \| \| \| \Delta ADE$ $\therefore \frac{AE}{AD} = \frac{AC}{AE}$ $\therefore AE^2 = AD \times AC$ $\therefore AE = \sqrt{AD \times AC}$	 Common proven ∠; ∠; ∠ from |||Δs	 ✓S ✓S ✓S ✓Answer (4)
			[12]